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Scheduling of Crude Oil Operations Under Demand Uncertainty: A Robust Optimization Framework Coupled with Global Optimization

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Scheduling of crude oil operations is an important component of overall refinery operations, because crude oil costs account for about 80% of the refinery turnover. The mathematical modeling of blending different crudes in storage tanks results in many bilinear terms, which transform the problem into a challenging, nonconvex, mixed-integer nonlinear programming (MINLP) optimization model. In practice, uncertainties are unavoidable and include demand fluctuations, ship arrival delays, equipment malfunction, and tank unavailability. In the presence of these uncertainties, an optimal schedule generated using nominal parameter values may often be suboptimal or even become infeasible. In this article, the robust optimization framework proposed by Lin et al. and Janak et al. is extended to develop a deterministic robust counterpart optimization model for demand uncertainty. The recently proposed branch and bound global optimization algorithm with piecewise-linear underestimation of bilinear terms by Li et al. is also extended to solve the nonconvex MINLP deterministic robust counterpart optimization model and generate robust schedules. Two examples are used to illustrate the capability of the proposed robust optimization approach, and the extended branch and bound global optimization algorithm for demand uncertainty. The computational results demonstrate that the obtained schedules are robust in the presence of demand uncertainty. © 2011 American Institute of Chemical Engineers *AIChE J,* 58: 2373–2396, 2012

Keywords: refinery, crude oil scheduling, mixed-integer nonlinear programming, nonconvex, global optimality, robust optimization

Introduction

In recent years, refineries have to exploit all potential cost-saving alternatives because of their intense competition arising from fluctuating product demands, ever-changing crude prices, and strict environmental regulations. Scheduling of crude oil operations is a critical component of the overall refinery operations,[1–3] because crude oil costs can account for about 80% of the refinery turnover.[4] Most refineries blend premium crudes with low-quality crudes over time to exploit the higher profit margins of low-quality crudes. However, the low-cost crudes can lead to processing problems in crude distillation units (or CDUs) and downstream units, because they usually contain some less-than-desirable properties with high composition. Therefore, a key issue is to exploit blends of low-cost crudes and premium crudes to maximize profit margins and minimize the operational problems at the same time. Optimal scheduling of crude oil operations using advanced mathematical optimization techniques such as mixed-integer linear programming (MILP) can increase profit margins using cheaper crudes, minimizing crude changeovers, avoiding ship demurrage, and managing crude inventories. The presence of crude blending gives rise to bilinear terms in the mathematical formulation for scheduling, while discrete scheduling decisions such as selecting a tank to unload or feed and the often complex nonlinear nature of crude properties and qualities make such a model challenging, nonlinear, nonconvex mixed-integer nonlinear problem (MINLP).

The crude oil scheduling problem has received considerable attention with researchers developing different models based on discrete- and continuous-time representations.[3] Li et al.[3] recently developed a novel unit-specific event-based continuous-time[5–21] MINLP formulation for this problem. They incorporated many realistic operational features such as single buoy mooring (SBM), multiple jetties, multiparcel vessels, single-parcel vessels, crude blending, brine settling, crude segregation, and multiple tanks feeding one CDU at one time and vice versa. In addition, 15 important volume-based or weight-based crude property indices were considered. To address this nonconvex MINLP problem, they exploited recent advances in piecewise-linear underestimation of bilinear terms[22–32] within a branch and bound algorithm for global optimization. The proposed model significantly reduced the number of bilinear terms and problem size compared to the discrete-time formulation of Reddy

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et al.[33] and Li et al.[34] The computational results showed that the proposed branch and bound global optimization algorithm with piecewise-linear underestimation[22–32] of the bilinear terms was effective to address all tested examples and resulted in better integer feasible solutions. More importantly, these integer feasible solutions were guaranteed to be within 2% of global optimality.

All of the aforementioned models assume that the parameters used in the models are deterministic in nature. However, frequent uncertainties in practice are unavoidable such as demand fluctuations, ship arrival delays, crude quality specification variations, uncertainty on crude profit margin, demurrage cost, inventory cost, changeover cost and safety stock penalty, and equipment malfunction, and tank unavailability. In the presence of these uncertainties, an optimal crude schedule obtained using nominal parameter values may often be suboptimal or even become infeasible. Different methodologies can be used to address this problem. In general, there are two approaches to address uncertainties: reactive scheduling and preventive scheduling.[35] Reactive scheduling is a process to revise the generated schedule from nominal parameters when a disruption has occurred during the actual execution of the schedule. Equipment malfunction and tank unavailability are events which are best modeled after realization by means of reactive scheduling techniques. Because of the ''on-line'' nature of reactive scheduling, an updated schedule from the nominal schedule must be generated in a timely manner and hence heuristic approaches are often utilized.[36,37] Preventive scheduling seeks to accommodate future uncertainty at the scheduling stage. The uncertainty related to demand, ship arrival time, crude specification, crude profit margin, demurrage cost, inventory cost, changeover cost, and safety stock penalty can be explicitly taken into account through preventive approaches such as two-stage stochastic programming, parametric programming, fuzzy programming, chance constraint programming, robust optimization techniques, and risk mitigation techniques.[35] Among these approaches, uncertain parameters are often represented by scenarios or nonscenarios. For detailed reviews on planning and scheduling under uncertainty, the reader is directed to Li and Ierapetritou,[38] and Verderame et al.[35]

Robust optimization focuses on developing preventive models to minimize the effects of uncertainties on the performance measure such as profit and operating cost. Its main objective is to ensure that the generated solutions are robust, while maintaining a high level of solution quality. Li et al.[39] developed scenario-based models for demand and ship arrival uncertainties separately and obtained more robust schedules compared to the nominal schedules. However, the number of scenarios exponentially increases with the number of uncertain parameters and hence makes their model intractable for practical problems with large number of uncertain parameters. Cao et al.[40] proposed an optimization model based on chance-constrained programming to generate robust schedules under demand uncertainty during scheduling of crude oil operations. Their approaches avoided the drawback of enumerating scenarios. However, their approach cannot be used to deal with uncertain parameters following a discrete probability distribution.[41] More importantly, their approach results in composition discrepancy. Recently, Wang and Rong[41] developed a two-stage robust optimization model for crude oil scheduling problem to address demand and ship arrival uncertainty separately. In their model, uncertain product demands were represented by chance-constrained programming and fuzzy programming, and ship arrival delay was represented by a scenario approach. Although their model can cope with a wide variety of uncertainties, the generated schedule from their model also results in composition discrepancy.

The above approaches cannot ensure the generated solution to be feasible for the nominal parameters. To overcome this disadvantage, Ben-Tal, Nemirovski, and coworkers[42–45] developed the robust optimization framework to handle uncertain parameters within linear and quadratic programming problems. A similar robust optimization framework was also independently proposed by Ghaoui and coworkers.[46,47] In their framework, various forms of parameter uncertainties are explicitly addressed and the robust solution is guaranteed to be feasible for the nominal parameters. Lin et al.,[48] Janak et al.,[49] Verderame and Floudas,[50–52] and Li et al.[53] extended and developed the theory of the robust optimization framework for general MILP problems with bounded, bounded and symmetric, and several known probability distributions. Also, Bertsimas and coworkers[54–56] extended and applied a robust optimization framework for linear and discrete programming. Recently, the robust optimization framework has been successfully extended and applied to addressing demand due date and demand amount uncertainty[50] in the problem of operational planning of large-scale industrial bath plants,[57] demand and processing time uncertainty[51] in the problem of integration of operational planning and medium-term scheduling for large-scale industrial batch plants,[57] and demand and transportation uncertainty[52] in multisite planning problem.[56] In addition, they applied an alternative framework based on conditional value-at-risk theory to address demand due date and demand amount uncertainty[58] in the operational planning of batch processes,[44] and demand and transportation uncertainty in the operational planning of multisite batch plants.[59]

To the best of our knowledge, the robust optimization framework has not yet been extended and applied for crude oil scheduling operations under uncertainty. In this article, we address the crude oil scheduling problem described by Li et al.[3] for a typical marine-access refinery under demand uncertainty. The unit-specific event-based continuous-time formulation developed by Li et al.[3] is used as the basis. The theory of robust optimization framework is used to develop the robust counterpart optimization model where a new approach is proposed to convert demand equality constraints to inequalities. Then, the branch and bound global optimization algorithm from Li et al.[3] is extended to solve the proposed deterministic robust counterpart optimization model and generate robust schedules. The computational results show that the schedule obtained from the proposed deterministic robust counterpart is robust in the presence of demand uncertainty.

Problem Statement

Consider Figure 1, which shows a schematic of crude oil unloading, storage and processing in a typical marine-access refinery. It involves offshore facilities for crude unloading such as a SBM station, onshore facilities for crude unloading such as B jetties, I (i = 1, 2, 3,…,I) crude storage tanks, and U (u = 1, 2, 3,…,U) CDUs. The pipeline connecting the SBM station with crude tanks is called the SBM line, and it normally has a substantial holdup. In this study, we assume that the refinery has no separate charging tanks, and hence crude storage tanks also act as charging tanks. Very large

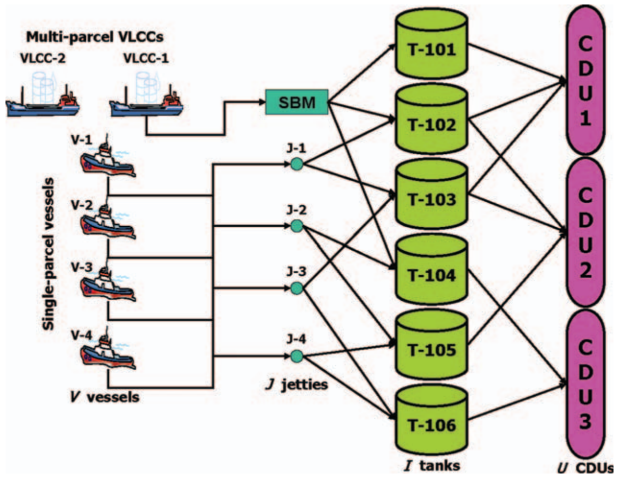

Figure 1. Schematic of crude oil unloading, blending, and processing.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

crude carriers (VLCCs) carrying multiparcels can dock at the SBM station and unload crudes into storage tanks. Single-parcel vessels carrying single crude for each can berth at jetties and unload crudes into storage tanks. Different types of crudes C ($c = 1, 2, 3,\ldots,C$) can be allowed to blend in these crude storage tanks. After blending, they are fed into CDUs for processing.

Given:

1. V ships, their expected arrival times, their crude parcels, and parcel sizes;
2. B Jetties, jetty-tank and SBM-tank connections, crude unloading transfer rates, and SBM pipeline holdup volume and its resident crude;
3. I storage tanks, their capacities, their initial crude volumes and compositions, and crude quality specifications or limits;
4. U CDUs, their processing rates, and crude quality specifications or limits;
5. Scheduling horizon H and product demands;
6. Economic data: crude margins, demurrage, crude changeover costs, and safety stock penalties.

Determine:

1. Unloading schedule for each ship including the timings, rates, and tanks for all parcel transfers;
2. Inventory and crude concentration profiles of all storage tanks;
3. Charging schedule for each CDU including the feed tanks, feed rates, and timings.

Subject to the operating practices:

1. Only one VLCC can dock at the SBM station at a time.
2. The unloading sequence of VLCC parcels is known *a priori*.
3. A parcel can unload to at most one storage tank at any time, but may unload to multiple tanks over time.
4. Each tank needs 8 h to settle and remove brine after each crude receipt.
5. A storage tank cannot receive and feed simultaneously.
6. Multiple tanks can feed a CDU simultaneously, and vice versa.

Assumptions:

1. Only one crude resides in the SBM line at the end of each parcel transfer. Crude flow is plug flow in the SBM.
2. All jetties are identical.
3. Holdup of the jetty pipeline is negligible.
4. Crude mixing is perfect in each storage tank.
5. Crude changeover times are negligible.
6. During operation, CDUs never shut down.

The objective is to maximize the gross profit, which is the revenue computed in terms of crude margins minus the operating costs such as demurrage and safety stock penalties.

During the above crude oil scheduling operations, frequent uncertainties are unavoidable such as demand fluctuations and ship arrival delays. The most common uncertainty arises from: (1) demand, (2) ship arrival, (3) crude quality specifications, and (4) some economic coefficients. These uncertain parameters can be described using discrete or continuous distributions. In some cases, only limited knowledge about the distribution is available, for example, the uncertainty is bounded, or the uncertainty is symmetrically distributed in a certain range. In the best situation, the distribution function for the uncertain parameter is given, for instance, as a normal distribution with known mean and standard deviation. In this article, we focus on demand uncertainty which is considered as: (a) bounded, (b) symmetrical and bounded, (c) following a known distribution such as normal distribution, or (d) following an unknown probability distribution. It should be noted that the random demand parameters considered in this article are assumed to be fully independent.

Deterministic Mathematical Formulation for Scheduling of Crude Oil Operations and Branch and Bound Global Optimization Algorithm

The model of Li et al.[3] was developed based on unit-specific event-based continuous-time representation,[5–21] which is different from other variants such as discrete-time,[5,9] process slots,[2] and unit slots.[60–62] The differences among those time representations are discussed by Floudas and Lin[5,6] and Li et al.[63] In the model of Li et al.,[3] they defined jetties, storage tanks (i), and CDUs (u) as units (m), and treated all identical jetties as one single resource. For each unit m, the scheduling horizon $[0, H]$ is divided into N ($n = 1, 2,\ldots,N$) event points (Figure 2). They also defined two binary variables to denote parcel-to-tank and tank-to-CDU connections, respectively.

$$X(p,i,n) = \begin{cases} 1 & \text{if parcel } p \text{ is unloaded to} \\ & \text{tank } i \text{ during event point } n \quad \forall (p,i) \in \mathbf{S}_{P,I} \\ 0 & \text{otherwise} \end{cases}$$

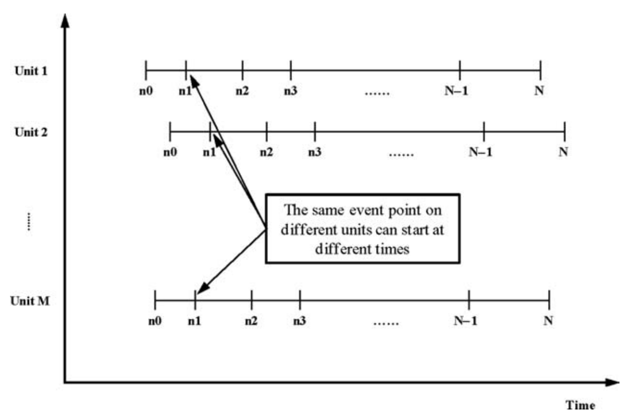

Figure 2. Event points definition for each unit.

Table 1. Data for Example 1

Tanker	Arrival Time	Parcel No: (Crude, Parcel Size, kbbl)
VLCC-1	0	1: (C2, 10) 2: (C1, 300) 3: (C4, 300) 4: (C3, 340)

				Initial Crude Composition (kbbl)		Crude Concentration Range in Tanks	
Tank	Initial Inventory (kbbl)	Capacity (kbbl)	Heel (kbbl)	C1 or C3	C2 or C4	C1 or C3 Min-Max	C2 or C4 Min-Max
T1	300	700	50	200	100	0–1	0–1
T2	300	700	50	100	200	0–1	0–1
T3	200	700	50	50	150	0–1	0–1
T4	300	700	50	130	170	0–1	0–1
T5	80	700	50	50	30	0–1	0–1

	Range of Crude Concentration for CDUs						
CDU	C1 Min–Max	C2 Min–Max	C3 Min–Max	C4 Min–Max	Key Comp.Range Min–Max	Range of Demand/8 h Min–Max	Demand (kbbl)
CDU1	0–1	0–1	0–0	0–0	0.0045–0.006	50–100	600
CDU2	0–0	0–0	0–1	0–1	0.014–0.0153	50–100	600

Flow Rate Limit (kbbl/8 h)							
Parcel-Tank Min–Max	Tank-CDU Min–Max	Demurrage Cost (k$/8 h)	Changeover Loss (k$/instance)	Safe Inventory Penalty ($/bbl/8 h)	Crude	Concentration of Key Composition	Margin ($/bbl)
10–400	0–100	100	5	0.2	C1	0.005	3
Tanks 1, 4 store crude 1–2 (Class 1); 2–3, 5 store crude 3–4 (Class 2)					C2	0.006	4.5
CDU 1 processes crudes 1–2; 2 processes crudes 3–4					C3	0.0165	5
The desire safety stock is 1200 kbbl					C4	0.0145	6

$$Y(i,u,n) = \begin{cases} 1 & \text{if tank } i \text{ feeds CDU } u \\ & \text{during event point } n \quad \forall (i,u) \in \mathbf{S}_{I,U} \\ 0 & \text{otherwise} \end{cases}$$

where, $\mathbf{S}_{P,I} = \{(p, i)|$ parcel p that can be unloaded to tank $i\}$ and $\mathbf{S}_{I,U} = \{(i, u)|$ tank i that can feed CDU $u\}$.

The complete nonconvex MINLP model is presented in Appendix A. For a detailed discussion of the model, the reader is referred to Li et al.[3]

Basic Components of Branch and Bound Global Optimization Algorithm

At each node in the branch and bound tree, a piecewise-linear relaxation of the node is minimized, and the node is branched to create two child nodes. After solving the piecewise-linear relaxation, a pool of feasible solutions (denoted as Pool-1) including the final solve (which is the best or optimal integer solution for the relaxation) is obtained. The lower bound (LB) is updated with this final solve, if the final solve is greater than the current LB. Each solution from the Pool-1 is used to fix the current values of the binary variables, initialize the continuous variables, and locally minimize the resulting NLP. All generated feasible local optimal solutions from another pool denoted as Pool-2. If the smallest objective value in the Pool-2 is less than the current upper bound (UB), then UB is updated with this value. At each step, the nodes with relaxations within a predetermined tolerance (denoted as ε) of the current UB are eliminated. The algorithm terminates with ε-convergence. Comprehensive coverage of the branch and bound algorithms can be found in the textbooks of Floudas.[64,65] Appendix B presents an outline of the strategies that are used in the branch and bound global optimization algorithm, which include piecewise-linear underestimators, branching strategy, solution improvement strategy, optimality-based tightening LB and UB, and so on. The details about these strategies can also be found in Li et al.[3]

Motivating example

Consider Example 1 involving one SBM pipeline, five storage tanks (T1–T5), and two CDUs (CDU101–CDU102). One VLCC carrying three crude parcels (300 kbbl C1, 300 kbbl C4, and 350 kbbl C3, unloaded in that sequence) arrives at time zero. At time zero, the SBM pipeline is holding 10 kbbl C2 from the last parcel. The scheduling horizon is about 72 h (i.e., 3 days). The nominal demands for both CDUs are 600 kbbl. Table 1 presents the complete data.

We solve this example with the model of Li et al.[3] and the proposed branch and bound global optimization algorithm using GAMS 22.6/CPLEX 11.0.0 on Dell OPTIPLEX 960 of Intel® Xeon™ CPU 3.0 GHz with 2 GB RAM running Linux. The computational performance is given in Table 2. The best solution of −$ 5631.707K was obtained within 15.3 CPU seconds, and it is guaranteed to be within

Table 2. Model and Solution Statistics for Example 1

	Nominal Solution	Robust Solution for Bounded Uncertainty	Robust Solution for Normal Distribution
Event points	3	3	3
GR	7	7	7
Binary variables	45	45	45
Continuous variables	753	1007	1007
Constraints	1159	1825	1825
Bilinear terms	70	210	210
Obj (UB, K$)	−5631.707	−5614.974	−5614.974
LB (K$)	−5687.152	−5671.139	−5667.320
Gap (%)	0.97	0.99	0.92
CPU time (s)	15.3	72.7	72.7

$Y = 0.05$, $\varepsilon = 0.01$.

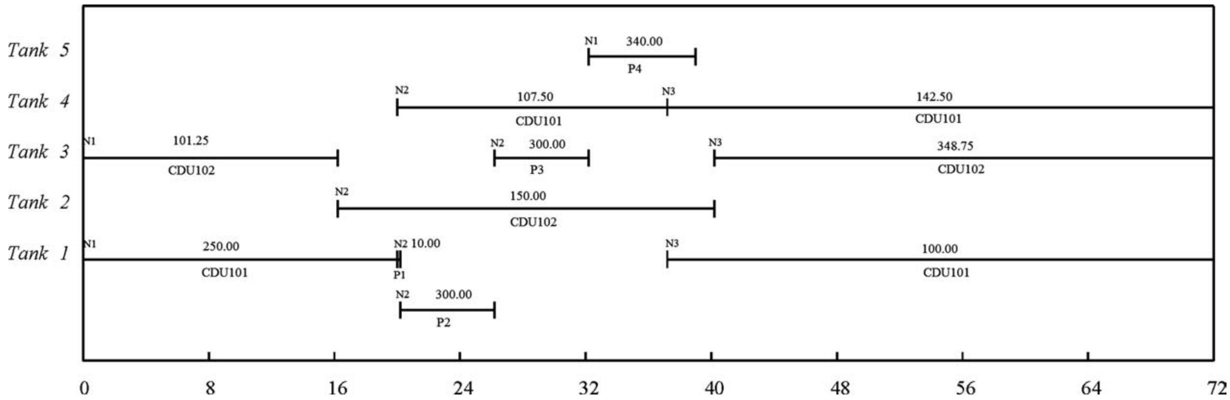

Figure 3. Nominal schedule for Example 1.

1% of global optimality. The operational schedule (i.e., nominal schedule) is illustrated in Figure 3. Figures 4 and 5 illustrate inventory profiles of tanks T1–T5 and feed rates to CDUs CDU101–102 from T1–T5.

Now let us consider an increase in the demands for both CDUs from the nominal value of 600 kbbl to 705 kbbl. We freeze the unloading schedule, tank-to-CDU connections, start and end times of tank feeding CDU operations from Figure 3 and solve the model again with the new demands (i.e., 705 kbbs for both CDUs). However, we cannot generate a feasible schedule. This reason is analyzed as follows. From Figure 3, T2 and T3 feed CDU102, and T5 does not. Although the feed rates to CDU102 do not reach its maximum feed rate (12.5 kbbl/s) at any time from Figure 5, the inventory level of T3 at $t = 72$ h reaches its minimum capacity (50 kbbl) from Figure 4. Thus, T3 cannot feed more crudes to CDU102. T2 can feed $(12.5-6.25) \times 24$ kbbl $=$ 150 kbbl additional crudes to CDU102. On the other hand, the inventory level of T2 at $t = 40.2$ h is 150 kbbl, which indicates that T2 can feed maximum 100 kbbl additional crudes to CDU102 during [16.2, 40.2] h. Hence, the maximum demand of CDU102 can reach 700 kbbl. In a brief, we cannot simply increase the feed rates to CDU102 from tanks T2, and T3 to meet its new demand of 705 kbbl.

If we do not freeze the unloading schedule, tank to CDU connections and those start and end times for tank feeding CDU operations from Figure 3, and the demands of

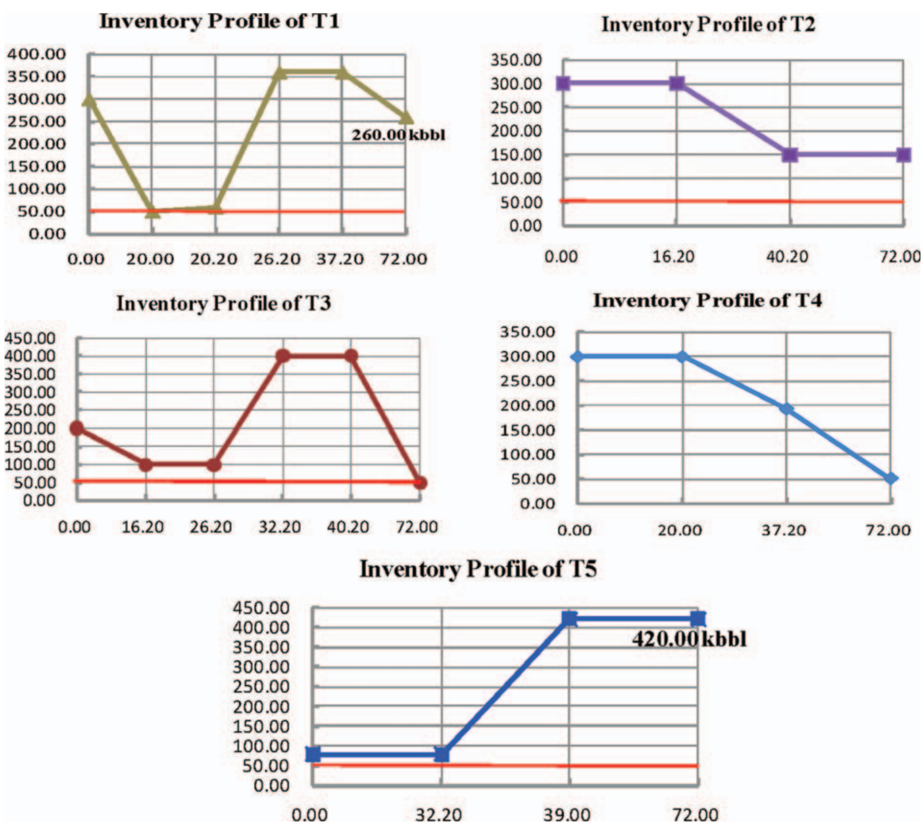

Figure 4. Inventory profiles of T1–T5 for Example 1 from the nominal schedule in Figure 3.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

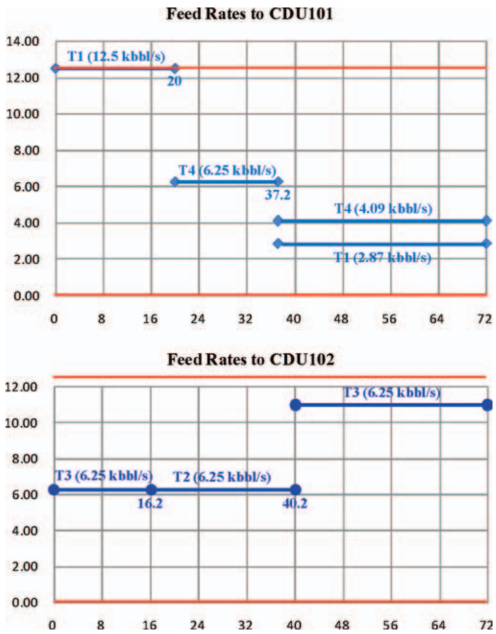

Figure 5. Feed rates of T1–T5 to CDU101 and CDU102 for Example 1 from the nominal schedule in Figure 3.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

CDU101 and CDU102 increase to 705 kbbl, then another new schedule is generated, which is illustrated in Figure 6. The new schedule in Figure 6 is much different from that in Figure 3. The differences may confuse the refiners and disrupt plant operation. In practice, it is desirable to keep the plant operation as close as possible to the nominal schedule (Figure 3). This example highlights the importance of generating reliable, efficient, and robust schedules and hence motivates us to develop systematic and effective techniques that generate robust schedules which may accommodate demand uncertainty.

Overview of the Robust Optimization Framework

The robust optimization framework developed by Lin et al.,[48] Janak et al.,[49] Verderame and Floudas,[50–52] and Li et al.[53] is used to address the various forms of uncertainty in this article. In the following, we present in brief the general robust optimization approach, which explicitly takes into account the various forms of parameter uncertainty within constraints and/or objective function.

Consider a generic deterministic MILP problem

$$\begin{aligned} \underset{x,y}{\mathrm{Min/Max}} \quad & c^T x + d^T y \\ s.t. \quad & Ex + Fy = e \\ & Ax + By \le p \\ & \underline{x} \le x \le \bar{x} \\ & y = 0, 1 \end{aligned} \tag{1}$$

Assume that the left-hand-side coefficients A and B and the right-hand-side parameters p of the inequality constraints are uncertain parameters. The true realization of an uncertain parameter is represented by

$$\tilde{a} = (1 + \varepsilon \cdot \xi) \cdot a \tag{2}$$

where a is an uncertain parameter with $\tilde{a}$ being its true realization, ε represents a give (relative) uncertain level, and ξ stands for a random variable.

In the robust optimization framework, a solution (x, y) is called robust if (1) (x, y) is feasible for the nominal problem, (2) whatever are the true values of the coefficients and right-hand-side parameters, (x, y) must satisfy the lth inequality constraint with an error of at most δ max[1, $|p_l|$], where δ is a given infeasibility tolerance. Then, any inequality constraint l in Eq. 1 becomes

$$\begin{aligned} \sum_{m \notin \mathbf{M}_l} a_{lm} x_m + \sum_{m \in \mathbf{M}_l} \tilde{a}_{lm} x_m + \sum_{k \notin \mathbf{K}_l} b_{lk} y_k \\ + \sum_{k \in \mathbf{K}_l} \tilde{b}_{lk} y_k \le \tilde{p}_l + \delta \max[1, |p_l|] \quad \forall l \end{aligned} \tag{3}$$

where $\mathbf{M}_l$ and $\mathbf{K}_l$ are the subsets that encompass those uncertain parameters for the given constraint l.

Bounded uncertainty

Assume that the uncertain parameters vary in a bounded interval. They are represented by

$$\begin{aligned} |\tilde{a}_{lm} - a_{lm}| &\le \varepsilon |a_{lm}| \\ |\tilde{b}_{lk} - b_{lk}| &\le \varepsilon |b_{lk}| \\ |\tilde{p}_l - p_l| &\le \varepsilon |p_l| \end{aligned} \tag{4}$$

The deterministic robust counterpart for Eq. 3 is derived as follows

$$\begin{aligned} \sum_m a_{lm} x_m + \sum_k b_{lk} y_k + \varepsilon \left(\sum_{m \in \mathbf{M}_l} |a_{lm}| u_m + \sum_{k \in \mathbf{K}_l} |b_{lk}| y_k \right) \\ \le p_l - \varepsilon |p_l| + \delta \max[1, |p_l|] \quad \forall l \end{aligned} \tag{5}$$

where $-u_m \le x_m \le u_m$.

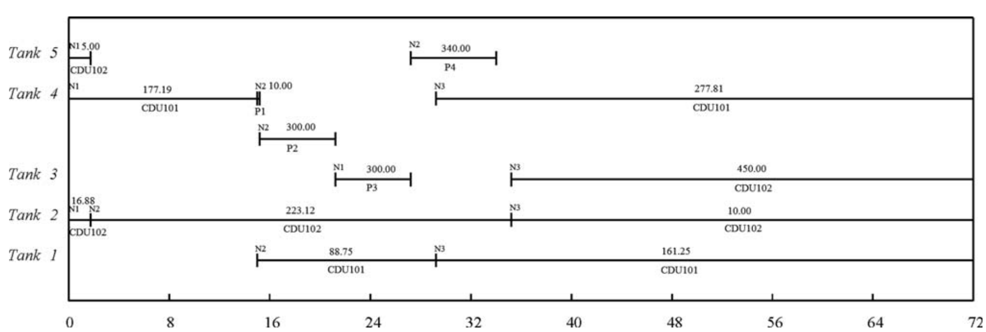

Figure 6. Operational schedule for Example 1 with demands of 705 kbbl.

The deterministic robust counterpart of the original uncertain MILP problem can be derived as follows

$$\begin{aligned}
&\underset{x,y,u}{\mathrm{Min/Max}} \quad c^T x + d^T y\\
&s.t. \qquad Ex + Fy = e\\
&\qquad\qquad Ax + By \le p\\
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k + \varepsilon\left(\sum_{m\in\mathbf{M}_l}|a_{lm}|u_m + \sum_{k\in\mathbf{K}_l}|b_{lk}|y_k\right) \qquad (6)\\
&\quad \le p_l - \varepsilon|p_l| + \delta\max[1,|p_l|] \qquad \forall l\\
&\qquad -u_m \le x_m \le u_m \qquad \forall m \in \mathbf{M}_l\\
&\qquad \underline{x} \le x \le \bar{x}\\
&\qquad y = 0,1 \qquad \forall k
\end{aligned}$$

Bounded and symmetric uncertainty

Assume that the uncertain parameters are distributed around their nominal values randomly and symmetrically represented as follows

$$\tilde{a}_{lm} = (1+\varepsilon\xi_{lm})a_{lm}, \quad \tilde{b}_{lk} = (1+\varepsilon\xi_{lk})b_{lk}, \quad \tilde{p}_l = (1+\varepsilon\xi_l)p_l \qquad (7)$$

where ξ_{lm}, ξ_{lk}, and ξ_l are random variables distributed symmetrically in the interval $[-1, 1]$.

The deterministic robust counterpart for Eq. 3 is derived as follows

$$\begin{aligned}
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k + \varepsilon\Bigg[\sum_{m\in\mathbf{M}_l}|a_{lm}|u_m + \sum_{k\in\mathbf{K}_l}|b_{lk}|w_{lk}\\
&\quad +|p_l|u0_l + \Omega\sqrt{\sum_{m\in\mathbf{M}_l}a_{lm}^2 z_{lm}^2 + \sum_{k\in\mathbf{K}_l}b_{lk}^2 v_{lk}^2 + p_l^2 z0_l^2}\Bigg]\\
&\qquad \le p_l + \delta\max[1,|p_l|] \quad \forall l \qquad (8)
\end{aligned}$$

where

$$\begin{aligned}
&-u_{lm} \le x_m - z_{lm} \le u_{lm} \qquad \forall\, m \in \mathbf{M}_l\\
&-w_{lk} \le y_k - v_{lk} \le w_{lk} \qquad \forall\, k \in \mathbf{K}_l\\
&-u0_t \le 1 + z0_t \le u0_t \qquad \forall\, l
\end{aligned}$$

The deterministic robust counterpart of the original uncertain MILP problem can be derived as follows

$$\begin{aligned}
&\underset{x,y,u,z,v,w,z0,u0}{\mathrm{Min/Max}} \quad c^T x + d^T y\\
&s.t. \qquad Ex + Fy = e\\
&\qquad\qquad Ax + By \le p\\
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k + \varepsilon\Bigg[\sum_{m\in\mathbf{M}_l}|a_{lm}|u_{lm} + \sum_{k\in\mathbf{K}_l}|b_{lk}|w_{lk}\\
&+|p_l|u0_l + \Omega\sqrt{\sum_{m\in\mathbf{M}_l}a_{lm}^2 z_{lm}^2 + \sum_{k\in\mathbf{K}_l}b_{lk}^2 v_{lk}^2 + p_l^2 z0_l^2}\Bigg]\\
&\qquad \le p_l + \delta\max[1,|p_l|] \qquad \forall l\\
&\qquad -u_{lm} \le x_m - z_{lm} \le u_m \qquad \forall m \in \mathbf{M}_l\\
&\qquad -w_{lk} \le y_k - v_{lk} \le w_{lk} \qquad \forall k \in \mathbf{K}_l\\
&\qquad -u0_l \le 1 + z0_l \le u0_l \qquad \forall l\\
&\qquad \underline{x} \le x \le \bar{x}\\
&\qquad y_k = 0,1 \qquad \forall k
\end{aligned}$$

where Ω is a positive parameter with $\kappa = \exp(-\Omega^2/2)$.

Known probability distribution

The true values of the uncertain parameters are obtained from their nominal values by random variables

$$\begin{aligned}
\tilde{a}_{lm} &= (1+\varepsilon\xi_{lm})a_{lm}\\
\tilde{b}_{lk} &= (1+\varepsilon\xi_{lk})b_{lk} \qquad (9)\\
\tilde{p}_l &= (1+\varepsilon\xi_l)p_l
\end{aligned}$$

Assume that the probability distributions of the random variables ξ_{lm}, ξ_{lk}, and ξ_l are known. Then, Eq. 3 becomes

$$\begin{aligned}
&\sum_{m\notin\mathbf{M}_l} a_{lm}x_m + \sum_{m\in\mathbf{M}_l} a_{lm}(1+\varepsilon\xi_{lm})x_m + \sum_{k\notin\mathbf{K}_l} b_{lk}y_k\\
&+\sum_{k\in\mathbf{K}_l} b_{lk}(1+\varepsilon\xi_{lk})y_k \le p_l(1+\varepsilon\xi_l) + \delta\max[1,|p_l|] \quad \forall l \quad (10)
\end{aligned}$$

Eq. 10 is assumed to be violated by the user-specified parameter κ $(0 \le \kappa \le 1)$. In other words

$$\begin{aligned}
&\Pr\Bigg\{\sum_m a_{lm}x_m + \sum_k b_{lk}y_k - p_l + \varepsilon\Bigg(\sum_{m\in\mathbf{M}_l} a_{lm}\xi_{lm}x_m\\
&+\sum_{k\in\mathbf{K}_l} b_{lk}\xi_{lk}y_k - p_l\xi_l\Bigg) > \delta\max[1,|p_l|]\Bigg\} \le \kappa \qquad \forall l \quad (11)
\end{aligned}$$

Note that if the tolerance for the constraint violation is low, then the value of κ should be similarly small.

We aggregate the respective random variables into one random variable.

$$\xi_l = \sum_{m\in\mathbf{M}_l} a_{lm}\xi_{lm}x_m + \sum_{k\in\mathbf{K}_l} b_{lk}\xi_{lk}y_k - p_l\xi_l \qquad \forall l \qquad (12)$$

We define F_{ξ_l} as the cumulative distribution for the aggregate random variable ξ_l, and $F_{\xi_l}^{-1}$ to be inverse cumulative distribution.

$$F_{\xi_l}(\lambda) = \Pr\{\xi_l \le \lambda\} = 1 - \kappa \qquad \forall l \qquad (13)$$

$$F_{\xi_l}^{-1}(1-\kappa) = f(\lambda, |a_{lm}|x_m, |b_{lk}|y_k, |p_l|) \qquad \forall l \qquad (14)$$

Combing Eqs. 12–14, the generic deterministic robust counterpart of the probabilistic constraint can be formulated as follows for random variables following any distribution.

$$\begin{aligned}
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k - p_l + \varepsilon\cdot f(\lambda, |a_{lm}|x_m, |b_{lk}|y_k, p_l)\\
&\qquad \le \delta\max[1,|p_l|] \qquad \forall l \qquad (15)
\end{aligned}$$

The deterministic robust optimization model for the original uncertain MILP problem can be derived as follows

$$
\begin{aligned}
&\text{Min/Max}\quad c^T x + d^T y \\
&s.t. \qquad Ex + Fy = e \\
&\qquad\qquad Ax + By \le p \\
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k - p_l + \varepsilon \cdot f(\lambda, |a_{lm}|x_m, |b_{lk}|y_k, p_l) \\
&\qquad\qquad\qquad \le \delta \max[1, |p_l|] \qquad \forall l \\
&\qquad \underline{x} \le x \le \bar{x} \\
&\qquad y_k = 0, 1 \qquad \forall k
\end{aligned}
$$

Unknown probability distribution

In this case, the right-hand side parameters in Eq. (1) are uncertain and follow an unknown probability distribution, i.e.

$$
\sum_m a_{lm}x_m + \sum_k b_{lk}y_k \le \sum_{i\in \mathbf{I}_l} \tilde{p}_{li} + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \quad \forall l \tag{16}
$$

where $p_l = \sum_i p_{li}$.

Its probabilistic form with constraint violation of the user-specified parameter κ is represented by

$$
\begin{aligned}
&\Pr\left\{ \sum_m a_{lm}x_m + \sum_k b_{lk}y_k > \sum_{i\in \mathbf{I}_l} \tilde{p}_{li} + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \right\} \le \kappa \qquad \forall l \\
\Rightarrow\quad &\Pr\left\{ -\sum_{i\in \mathbf{I}_l} \tilde{p}_{li} > -\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \right\} \le \kappa \quad \forall l
\end{aligned} \tag{17}
$$

From Markov Inequality,[50] the left-hand side of Eq. 17 is

$$
\Pr\left\{ -\sum_{i\in \mathbf{I}_l} \tilde{p}_{li} > -\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \right\} \le \frac{-E\left[\sum_{i\in \mathbf{I}_l} \tilde{p}_{li}\right]}{-\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|]} \tag{18}
$$

If the unknown distribution is approximately symmetric and the extremes are equidistant from the mean, for a given value of κ

$$
E\left[\sum_{i\in \mathbf{I}_l} \tilde{p}_{li}\right] \ge \max\left[0, 2\kappa \cdot \sum_{i\in \mathbf{I}_l} p_{li} - \sum_{i\in \mathbf{I}_l} p_{li}\right] \tag{19}
$$

Combine Eqs. 18 and 19

$$
\Pr\left\{ -\sum_{i\in \mathbf{I}_l} \tilde{p}_{li} > -\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \right\} \le \frac{-\max\left[0, 2\kappa \cdot \sum_{i\in \mathbf{I}_l} p_{li} - \sum_{i\in \mathbf{I}_l} p_{li}\right]}{-\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|]}
$$

If let $\frac{-\max\left[0, 2\kappa \cdot \sum_{i\in \mathbf{I}_l} p_{li} - \sum_{i\in \mathbf{I}_l} p_{li}\right]}{-\sum_m a_{lm}x_m - \sum_k b_{lk}y_k + \sum_{i\notin \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|]} \le \kappa$, then the deterministic robust counterpart for Eq. 16 is represented by

$$
\sum_m a_{lm}x_m + \sum_k b_{lk}y_k \le \sum_{i\notin \mathbf{I}_l} p_{li} + \max\left[0, \frac{2\kappa - 1}{\kappa}\right] \cdot \sum_{i\in \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \qquad \forall l \tag{20}
$$

The deterministic robust counterpart optimization model for the original uncertain MILP problem can be derived as follows

$$
\begin{aligned}
&\text{Min/Max}\quad c^T x + d^T y \\
&s.t. \qquad Ex + Fy = e \\
&\qquad\qquad Ax + By \le p \\
&\sum_m a_{lm}x_m + \sum_k b_{lk}y_k \le \sum_{i\notin \mathbf{I}_l} p_{li} + \max\left[0, \frac{2\kappa - 1}{\kappa}\right] \\
&\qquad\qquad\qquad \times \sum_{i\in \mathbf{I}_l} p_{li} + \delta \max[1, |p_l|] \qquad \forall l \\
&\qquad \underline{x} \le x \le \bar{x} \\
&\qquad y_k = 0, 1 \qquad \forall k
\end{aligned}
$$

Up to this point, the robust optimization framework has been reviewed for uncertain parameters following a bounded, bounded and symmetric distribution, known probability distribution, and unknown probability distribution. For simplicity, universal values for ε, κ, and δ are adopted. However, Janak et al.[49] highlighted that the robust optimization framework can easily be extended for cases where ε can vary from parameter to parameter and κ, and δ can vary from constraint to constraint. Next, we extend and apply the robust optimization framework to address the various forms of demand uncertainty present during crude oil scheduling operations.

Robust Counterpart for Demand Uncertainty during Scheduling of Crude Oil Operations

In the deterministic model of Li et al.,[3] the deterministic demand constraint is presented as follows

$$
\sum_n V_U(u, n) = D(u) \qquad \forall u \tag{21}
$$

where $V_U(u,n)$ is total amount of crudes fed to CDU u during event point n and $D(u)$ is the nominal demand of CDU u. Note that Eq. 21 features equalities. To apply the robust optimization framework, Eq. 21 must be converted to inequality.

We notice that Eq. 21 is equivalent to the following two constraints:

$$
\sum_n V_U(u, n) \le D(u) \qquad \forall u \tag{22}
$$

$$\sum_n V_U(u,n) \geq D(u) \qquad \forall u \tag{23}$$

If we apply the robust optimization techniques to Eqs. 22 and 23 simultaneously, then it is not difficult to show that we cannot generate any feasible solution to meet those robust counterparts and Eq. 21 at nominal value simultaneously. To avoid this, we define a new set **SC** (sc = 1 and 2) and new auxiliary variables $rV_U(u,n,\text{sc})$.

$$\sum_n rV_U(u,n,\text{sc}) \leq \tilde{D}(u) \qquad \forall u, \text{sc}=1 \tag{24a}$$

$$\sum_n rV_U(u,n,\text{sc}) \geq \tilde{D}(u) \qquad \forall u, \text{sc}=2 \tag{24b}$$

where $\tilde{D}(u)$ is the true realization of the uncertain demand parameter.

The robust optimization techniques can be easily applied to Eq. 24a,b simultaneously.

Bounded uncertainty

If the demand parameter is uncertain and varies in a bounded interval [D^L (u), D^U (u)], then the robust counterparts of Eq. 24a,b are given by

$$\sum_n rV_U(u,n,\text{sc}) \leq D^L(u) + \delta \max(1, |D(u)|) \qquad \forall u, \text{sc}=1 \tag{25a}$$

$$\sum_n rV_U(u,n,\text{sc}) \geq D^U(u) - \delta \max(1, |D(u)|) \qquad \forall u, \text{sc}=2 \tag{25b}$$

Bounded and symmetric uncertainty

If the demand parameters are uncertain and distributed around the nominal values randomly and symmetrically as follows

$$\tilde{D}(u) = [1 + \varepsilon \cdot \xi(u)] \cdot D(u)$$

where $\xi(u)$ are random variables distributed symmetrically in the interval [−1, 1].

The robust counterparts for Eq. 24a,b are derived as follows

$$\sum_n rV_U(u,n,\text{sc}) + \varepsilon \cdot D(u) \cdot [ru(u,\text{sc}) + \Omega \cdot rzz(u,\text{sc})] \leq D(u) + \delta \max[1, D(u)] \qquad \forall u, \text{sc}=1 \tag{26a}$$

where

$$\begin{aligned} &-ru(u,\text{sc}) \leq 1 + rz(u,\text{sc}) \leq ru(u,\text{sc}) && \forall u, \text{sc}=1 \\ &-rzz(u) \leq rz(u) \leq rzz(u) && \forall u, \text{sc}=1 \\ &rz(u) \geq 0, rzz(u) \geq 0 && \forall u, \text{sc}=1 \end{aligned}$$

$$\sum_n V_U(u,n,\text{sc}) \geq D(u) + \varepsilon \cdot D(u) \cdot [ru(u,\text{sc}) + \Omega \cdot rzz(u,\text{sc})] - \delta \max[1, D(u)] \qquad \forall u, \text{sc}=2 \tag{26b}$$

where

$$\begin{aligned} &-ru(u,\text{sc}) \leq 1 + rz(u,\text{sc}) \leq ru(u,\text{sc}) && \forall u, \text{sc}=2 \\ &-rzz(u) \leq rz(u) \leq rzz(u) && \forall u, \text{sc}=2 \\ &rz(u) \geq 0, rzz(u) \geq 0 && \forall u, \text{sc}=2 \end{aligned}$$

Note that Ω is a positive parameter with $\kappa = \exp(-\Omega^2/2)$ and the violation probability is 2κ.

Known probability distribution

The generic robust deterministic counterpart of the probabilistic constraint can be formulated as follows for random variables following any distribution.

$$\sum_n rV_U(u,n,\text{sc}) + \varepsilon \cdot f[\lambda, D(u)] \leq D(u) + \delta \max[1, D(u)] \qquad \forall u, \text{sc}=1 \tag{27a}$$

$$\sum_n rV_U(u,n,sc) \geq D(u) + \varepsilon \cdot f[\lambda, D(u)] - \delta \max[1, D(u)] \qquad \forall u, \text{sc}=2 \tag{27b}$$

Note that the violation probability is 2κ.

Uniform continuous distribution

If the demand parameters are uncertain and follow a uniform continuous distribution, then $f[\lambda, D(u)] = (1-2\kappa) \cdot D(u)$. The deterministic robust counterpart is represented by

$$\sum_n rV_U(u,n,\text{sc}) + \varepsilon \cdot (1-2\kappa)D(u) \leq D(u) + \delta \max[1, D(u)] \qquad \forall u, \text{sc}=1 \tag{28a}$$

$$\sum_n rV_U(u,n,\text{sc}) \geq D(u) + \varepsilon \cdot (1-2\kappa)D(u) - \delta \max[1, D(u)] \qquad \forall u, \text{sc}=2 \tag{28b}$$

Normal distribution

If the demand parameters follow a normal distribution, then $f[\lambda, D(u)] = F_n^{-1}(1-\kappa) \cdot D(u)$ where $F_n^{-1}(1-\kappa)$ being the inverse standard normal cumulative distribution function. The robust counterpart is represented by

$$\sum_n rV_U(u,n,\text{sc}) + \varepsilon \cdot F_n^{-1}(1-\kappa) \cdot D(u) \leq D(u) + \delta \max[1, D(u)] \qquad \forall u, \text{sc}=1 \tag{29a}$$

$$\sum_n rV_U(u,n,\text{sc}) \geq D(u) + \varepsilon \cdot F_n^{-1}(1-\kappa) \cdot D(u) - \delta \max[1, D(u)] \qquad \forall u, \text{sc}=2 \tag{29b}$$

The deterministic robust counterparts for other known probability distributions are also presented in Supporting Information, Appendix S1.

Unknown probability distribution

When the demand parameters are uncertain and follow an unknown probability distribution, then the robust counterparts for Eq. 24a,b are given by

$$\sum_n rV_U(u,n,\text{sc}) \geq \frac{1}{\kappa} D(u) - \delta \max[1, D(u)] \qquad \forall u, \text{sc}=1 \tag{30a}$$

$$\sum_n rV_U(u,n,\text{sc}) \leq \max\left[0, \frac{2\kappa - 1}{\kappa}\right] \cdot D(u) + \delta \max[1, D(u)] \qquad \forall u, \text{sc}=2 \tag{30b}$$

Note that the violation probability is 2κ.

Besides the above robust counterparts, we also define some other auxiliary variables including $rV_I(i,n,\text{sc})$, $rV_{I,C}(i,c,n,\text{sc})$, $rV_{I,U}(i,u,n,\text{sc})$, $rV_{I,U,C}(i,u,c,n,\text{sc})$, and $rE_{I,C}(i,c,n,\text{sc})$, and $rSSP(\text{sc})$. These variables are connected with each other and $rV_U(u,n,\text{sc})$ using the following additional constraints:

We first relate $rV_I(i,n,\text{sc})$ with the variable $rV_{I,C}(i,c,n,\text{sc})$. For each sc, the total crude volume in each tank i at the end of event point n is equivalent to summation of the volume of each crude c in this tank i.

$$\sum_{c:(i,c)\in \mathbf{S}_{I,C}} rV_{I,C}(i,c,n,\text{sc}) = rV_I(i,n,\text{sc}) \qquad \forall i,n,\text{sc} \qquad (31)$$

For each sc, at any time, the crude c in tank i must meet its lower $[E_I^{\min}(i,c)]$ and upper $[E_I^{\max}(i,c)]$ fractions in this tank.

$$rV_I(i,n,\text{sc})\cdot E_I^{\min}(i,c) \le rV_{I,C}(i,c,n,\text{sc}) \quad \forall (i,c)\in \mathbf{S}_{I,C},n,\text{sc} \qquad (32a)$$

$$rV_{I,C}(i,c,n,\text{sc}) \le rV_I(i,n,\text{sc})\cdot E_I^{\max}(i,c) \quad \forall (i,c)\in \mathbf{S}_{I,C},n,\text{sc} \qquad (32b)$$

Each tank i has several possible event points during which the concentration of this tank i is the same as its initial composition. During the possible event points

$$rV_{I,U,C}(i,u,c,n,\text{sc}) = E_I^{\text{init}}(i,c)\cdot rV_{I,U}(i,u,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},(i,c)\in \mathbf{S}_{I,C},(i,n)\in \mathbf{S}_{F,I},n,\text{sc} \qquad (33)$$

where $\mathbf{S}_{F,I}$ denotes the possible event points during which the concentration of this tank i is the same as its initial composition.

At other event point on each tank i [i.e., $(i, n) \notin \mathbf{S}_{F,I}$]

$$rV_{I,U,C}(i,u,c,n,\text{sc}) = rE_{I,C}(i,c,n-1,\text{sc})\cdot rV_{I,U}(i,u,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},(i,c)\in \mathbf{S}_{I,C},(i,n)\notin \mathbf{S}_{F,I},n,\text{sc} \qquad (34)$$

At the end of each event point n,

$$rV_{I,C}(i,c,n,\text{sc}) = rE_{I,C}(i,c,n,\text{sc})\cdot rV_I(i,n,\text{sc}) \quad \forall (i,c)\in \mathbf{S}_{I,C},n,\text{sc} \qquad (35)$$

For each sc, the total amount of crudes from storage tank i to CDU u during each event point n must meet its minimum $[F_{I,U}^{\min}(i,u)]$ and maximum $[F_{I,U}^{\max}(i,u)]$ feed rates.

$$rV_{I,U}(i,u,n,\text{sc}) \le F_{I,U}^{\max}(i,u)\cdot\left[T_{I,U}^{\text{end}}(i,u,n) - T_{I,U}^{\text{start}}(i,u,n)\right] \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (36a)$$

$$F_{I,U}^{\min}(i,u)\cdot\left[T_{I,U}^{\text{end}}(i,u,n) - T_{I,U}^{\text{start}}(i,u,n)\right] \le rV_{I,U}(i,u,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (36b)$$

For each sc, if tank i does not feed CDU u at event point n, then the total amount $[V_{I,U}(i,u,n)]$ charged should be zero.

$$rV_{I,U}(i,u,n,\text{sc}) \le V_{I,U}^{\max}(i,u,n)\cdot Y(i,u,n) \; \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (37)$$

The amount of crude c fed from tank i to CDU u during event point n for each sc $[V_{I,U,C}(i,u,c,n)]$ is computed by,

$$rV_{I,U}(i,u,n,\text{sc}) = \sum_{c:(i,c)\in \mathbf{S}_{I,C}} rV_{I,U,C}(i,u,c,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (38)$$

For each sc, the total amount of crudes $[V_U(u,n)]$ fed to each CDU u during event point n is given by,

$$\sum_{i:(i,u)\in \mathbf{S}_{I,U}} rV_{I,U}(i,u,n,\text{sc}) = rV_U(u,n,\text{sc}) \qquad \forall u,n,\text{sc} \qquad (39)$$

For each sc, the total amount of crudes fed to each CDU u during each event point n must meet its minimum $[D_U^{\min}(u)]$ and maximum $[D_U^{\max}(u)]$ processing rates.

$$D_U^{\min}(u)\left[T_U^{\text{end}}(u,n) - T_U^{\text{start}}(u,n)\right] \le rV_U(u,n,\text{sc}) \qquad \forall u,n,\text{sc} \qquad (40a)$$

$$rV_U(u,n,\text{sc}) \le D_U^{\max}(u)\left[T_U^{\text{end}}(u,n) - T_U^{\text{start}}(u,n)\right] \qquad \forall u,n,\text{sc} \qquad (40b)$$

The crude fraction in the feed to any CDU u under each sc must also meet its minimum $[E_U^{\min}(u,c)]$ and maximum $[E_U^{\max}(u,c)]$ fractions.

$$rV_U(u,n,\text{sc})\cdot E_U^{\min}(u,c) \le \sum_{i:(i,u)\in \mathbf{S}_{I,U}} rV_{I,U,C}(i,u,c,n,\text{sc}) \quad \forall (u,c)\in \mathbf{S}_{U,C},\text{sc} \qquad (41a)$$

$$\sum_{i:(i,u)\in \mathbf{S}_{I,U}} rV_{I,U,C}(i,u,c,n,\text{sc}) \le rV_U(u,n,\text{sc})\cdot E_U^{\max}(u,c) \quad \forall (u,c)\in \mathbf{S}_{U,C},\text{sc} \qquad (41b)$$

For every sc, the desired crude qualities feeding to CDUs must be ensured within the minimum $[e_U^{\min}(u,k)]$ and maximum $[e_U^{\max}(u,k)]$ acceptable limits on properties that in the feed to CDU u.

$$e_U^{\min}(u,k)\cdot\sum_{i:(i,u)\in \mathbf{S}_{I,U}} rV_{I,U}(i,u,n,\text{sc}) \le \sum_{i:(i,u)\in \mathbf{S}_{I,U}}\sum_{c:(i,c)\in \mathbf{S}_{I,C}} e_C(c,k)\cdot rV_{I,U,C}(i,u,c,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (42a)$$

$$\sum_{i:(i,u)\in \mathbf{S}_{I,U}}\sum_{c:(i,c)\in \mathbf{S}_{I,C}} e_C(c,k)\cdot rV_{I,U,C}(i,u,c,n,\text{sc}) \le e_U^{\max}(u,k)\cdot\sum_{i:(i,u)\in \mathbf{S}_{I,U}} rV_{I,U}(i,u,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (42b)$$

$$e_U^{\min}(u,k)\cdot\left(\sum_{i:(i,u)\in \mathbf{S}_{I,U}}\sum_{c:(i,c)\in \mathbf{S}_{I,C}} \rho_c\cdot rV_{I,U,C}(i,u,c,n,\text{sc})\right) \le \sum_{i:(i,u)\in \mathbf{S}_{I,U}}\sum_{c:(i,c)\in \mathbf{S}_{I,C}} e_C(c,k)\cdot\rho_c\cdot rV_{I,U,C}(i,u,c,n,\text{sc}) \quad \forall (i,u)\in \mathbf{S}_{I,U},\text{sc} \qquad (43a)$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}}\sum_{c:(i,c)\in\mathbf{S}_{I,C}} e_C(c,k)\cdot\rho_c\cdot rV_{I,U,C}(i,u,c,n,\mathrm{sc})$$
$$\le e_U^{\max}(u,k)\cdot\left(\sum_{i:(i,u)\in\mathbf{S}_{I,U}}\sum_{c:(i,c)\in\mathbf{S}_{I,C}}\rho_c\cdot rV_{I,U,C}(i,u,c,n,\mathrm{sc})\right)\quad\forall(i,u)\in\mathbf{S}_{I,U},\mathrm{sc}\tag{43b}$$

where, $e_C(c,k)$ denotes the known blending index for a property k of crude c, and ρ_c is the density of crude c.

The inventory balance for each storage tank i at the end of event point n for each sc can be expressed as follows,

$$rV_{I,C}(i,c,n,\mathrm{sc}) = rV_{I,C}(i,c,n-1,\mathrm{sc}) + \sum_{p:(p,c)\in\mathbf{S}_{P,C}} V_{P,I}(p,i,n)\times E_P(p,c) - \sum_{u:(i,u)\in\mathbf{S}_{I,U}} rV_{I,U,C}(i,u,c,n,\mathrm{sc})\quad\forall(i,c)\in\mathbf{S}_{I,C},n>1,\mathrm{sc}\tag{44a}$$

$$rV_{I,C}(i,c,n,\mathrm{sc}) = V_{I,C}^{\mathrm{init}}(i,c) + \sum_{p:(p,c)\in\mathbf{S}_{P,C}} V_{P,I}(p,i,n)\cdot E_P(p,c) - \sum_{u:(i,u)\in\mathbf{S}_{I,U}} rV_{I,U,C}(i,u,c,n,\mathrm{sc})\quad\forall(i,c)\in\mathbf{S}_{I,C},n=1,\mathrm{sc}\tag{44b}$$

The average safety stock for each sc [$SSP(\mathrm{sc})$] is calculated as,

$$rSSP(\mathrm{sc}) \ge SS - \frac{\sum_i\left(\sum_n rV_I(i,n,\mathrm{sc}) + V_I^{\mathrm{init}}(i)\right)}{N+1}\quad\forall\ \mathrm{sc}\tag{45}$$

where, SS is the desired safety stock of crude.

The hard bounds for those auxiliary variables are given below:

$$rSSP(\mathrm{sc}) \le SS\quad\forall\ \mathrm{sc}\tag{46}$$

$$0\le rE_{I,C}(i,c,n,\mathrm{sc})\le 1\quad\forall(i,c)\in\mathbf{S}_{I,C},\mathrm{sc}\tag{47a, b}$$

$$V_I^{\min}(i)\le rV_I(i,n,\mathrm{sc})\le V_I^{\max}(i)\quad\forall(i,c)\in\mathbf{S}_{I,C},\mathrm{sc}\tag{48a, b}$$

$$rV_{I,U}(i,u,n,\mathrm{sc}) \le \begin{cases}\min\left\{F_{I,U}^{\max}(i,u)\cdot H, V_I^{init}-V_I^{\min}(i,n), V_I^{\max}(i)-V_I^{\min}(i)\right\} & \text{if } n=1\\ \min\left\{\begin{array}{l}F_{I,U}^{\max}(i,u)\cdot H, V_I^{\max}(i,n-1)-V_I^{\min}(i,n)\\ V_I^{\max}(i)-V_I^{\min}(i)\end{array}\right\} & \text{if } n>1\end{cases}\quad\forall(i,u)\in\mathbf{S}_{I,U},\mathrm{sc}\tag{49}$$

The deterministic robust counterpart optimization model is denoted as **ROM** presented below.

(**ROM**) **Min** −PROFIT
s.t. Eqs. A1−A43 in **Appendix A**
Eqs. 25a, b, 26a, b, 27a, b, or 30a, b for corresponding distribution
Eqs. 31−45, and 46−49
Eqs. A45−A54 and Eqs. A55−A61 in **Appendix A**

The resulting mathematical model ROM is a nonconvex MINLP, and the sources of nonconvexities are the distinct bilinear terms (i.e., Eqs. A15, A16, 34 and 35)

$$V_{I,U,C}(i,u,c,n) = E_{I,C}(i,c,n-1)\cdot V_{I,U}(i,u,n)\quad\forall(i,u)\in\mathbf{S}_{I,U},(i,c)\in\mathbf{S}_{I,C},(i,n)\notin\mathbf{S}_{F,I},n\tag{A15$'$}$$

$$V_{I,C}(i,c,n) = E_{I,C}(i,c,n)\cdot V_I(i,n)\quad\forall(i,c)\in\mathbf{S}_{I,C},n\tag{A16$'$}$$

$$rV_{I,U,C}(i,u,c,n,\mathrm{sc}) = rE_{I,C}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}(i,u,n,\mathrm{sc})\quad\forall(i,u)\in\mathbf{S}_{I,U},(i,c)\in\mathbf{S}_{I,C},(i,n)\notin\mathbf{S}_{F,I},n,\mathrm{sc}\tag{34$'$}$$

$$rV_{I,C}(i,c,n,\mathrm{sc}) = rE_{I,C}(i,c,n,\mathrm{sc})\cdot rV_I(i,n,\mathrm{sc})\quad\forall(i,c)\in\mathbf{S}_{I,C},n,\mathrm{sc}\tag{35$'$}$$

Remarks: Cao et al.[40] and Wang and Rong[41] used only Eq. 23 to convert Eq. 21 to inequality. The corresponding deterministic robust counterparts with different distributions from the robust framework are presented in Supporting Information, Appendix S2. The deterministic robust counterpart optimization model of Cao et al.[40] and Wang and Rong[41] (ROM-CWR) always maximizes the total amount fed to each CDU for the objective of profit maximization regardless of specific demands of CDUs, as illustrated later.

Global Optimization Approach

In the model ROM, additional bilinear terms (i.e., Eqs. 34,35 or 34′,35′) are introduced. These bilinear terms are relaxed using only the McCormick convex and concave envelopes. The McCormick convex and concave envelopes for the bilinear terms (i.e., Eqs. 34, 35 or 34′, 35′) in the model ROM are derived as follows

$$rV_{I,U,C}(i,u,c,n,\mathrm{sc}) \begin{cases} \geq rE_{I,C}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\min}(i,u,n,\mathrm{sc}) + rE_{I,C}^{\min}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}(i,u,n,\mathrm{sc}) - rE_{I,C}^{\min}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\min}(i,u,n,\mathrm{sc}) \\ \leq rE_{I,C}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\min}(i,u,n,\mathrm{sc}) + rE_{I,C}^{\max}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}(i,u,n,\mathrm{sc}) - rE_{I,C}^{\max}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\min}(i,u,n,\mathrm{sc}) \\ \leq rE_{I,C}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\max}(i,u,n,\mathrm{sc}) + rE_{I,C}^{\min}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}(i,u,n,\mathrm{sc}) - rE_{I,C}^{\min}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\max}(i,u,n,\mathrm{sc}) \\ \geq rE_{I,C}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\max}(i,u,n,\mathrm{sc}) + rE_{I,C}^{\max}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}(i,u,n,\mathrm{sc}) - rE_{I,C}^{\max}(i,c,n-1,\mathrm{sc})\cdot rV_{I,U}^{\max}(i,u,n,\mathrm{sc}) \end{cases}$$

$$\forall (i,u) \in \mathbf{S}_{I,U}, (i,c) \in \mathbf{S}_{I,C}, n \quad (50)$$

$$rV_{I,C}(i,c,n,\mathrm{sc}) \begin{cases} \geq rE_{I,C}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\min}(i,n,\mathrm{sc}) + rE_{I,C}^{\min}(i,c,n,\mathrm{sc})\cdot rV_{I}(i,n,\mathrm{sc}) - rE_{I,C}^{\min}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\min}(i,n,\mathrm{sc}) \\ \leq rE_{I,C}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\min}(i,n,\mathrm{sc}) + rE_{I,C}^{\max}(i,c,n,\mathrm{sc})\cdot rV_{I}(i,n,\mathrm{sc}) - rE_{I,C}^{\max}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\min}(i,n,\mathrm{sc}) \\ \leq rE_{I,C}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\max}(i,n,\mathrm{sc}) + rE_{I,C}^{\min}(i,c,n,\mathrm{sc})\cdot rV_{I}(i,n,\mathrm{sc}) - rE_{I,C}^{\min}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\max}(i,n,\mathrm{sc}) \\ \geq rE_{I,C}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\max}(i,n,\mathrm{sc}) + rE_{I,C}^{\max}(i,c,n,\mathrm{sc})\cdot rV_{I}(i,n,\mathrm{sc}) - rE_{I,C}^{\max}(i,c,n,\mathrm{sc})\cdot rV_{I}^{\max}(i,n,\mathrm{sc}) \end{cases}$$

$$\forall (i,c) \in \mathbf{S}_{I,C}, n \quad (51)$$

The piecewise-linear relaxation of the model ROM denoted as RROM is defined as follows

(**RROM**) **Min** −PROFIT
s.t. Eqs. A1−A14, A17−A43 in **Appendix A**
Eqs. B1−B2 in **Appendix B**
Eqs. 25a, b, 26a, b, 27a, b, or 30a, b for corresponding distribution
Eqs. 31−33, 36−45, 46−49, and 50−51
Eqs. A45−A54 and Eqs. A55−A61 in **Appendix A**

Note that we do not use piecewise-linear relaxation for those variables $rE_{I,C}(i,c,n,\mathrm{sc})$, $rV_I(i,n,\mathrm{sc})$, and $rV_{I,U}(i,u,n,\mathrm{sc})$ in the model RROM.

A pool of feasible solutions including the final solve from the model RROM is used to fix the current values of the binary variables, initialize the continuous variables using their current values, and locally minimize the resulting NLP. The objective function for NLP changes to the following

$$\begin{aligned} \mathrm{PROFIT} = &\sum_i \sum_u \sum_c \sum_n C_{\mathrm{PROF}}(c)\cdot V_{I,U,C}(i,u,c,n) \\ &- \sum_v T_{CW}(v) - PEN\cdot SSP \cdot H \\ &- \sum_u \sum_n C_{\mathrm{SET}}\cdot z(u,n) \\ &+ \sum_i \sum_u \sum_c \sum_n \sum_{\mathrm{sc}} C_{PROF}(c)\cdot rV_{I,U,C}(i,u,c,n,\mathrm{sc}) \\ &- \sum_{\mathrm{sc}} PEN \cdot rSSP(\mathrm{sc})\cdot H \end{aligned} \quad (52)$$

The LP minimization and maximization problems for tightening the lower and UBs for variables $E_{I,C}(i,c,n)$, $V_I(i,n)$, and $V_{I,U}(i,u,n)$ can be stated as follows

(**ROML**) **Min** z_obbt
s.t. Eqs. A1−A14, A17−A43 in **Appendix A**
Eqs. B3−B4 in **Appendix B**
Eqs. 25a, b, 26a, b, 27a, b, or 30a, b for corresponding distribution
Eqs. 31−33, 36−45, 46−49, and 50−51
Eqs. A45−A54 and Eqs. A55−A61 in **Appendix A**
$0 \leq X(p,i,n), Y(i,u,n) \leq 1$

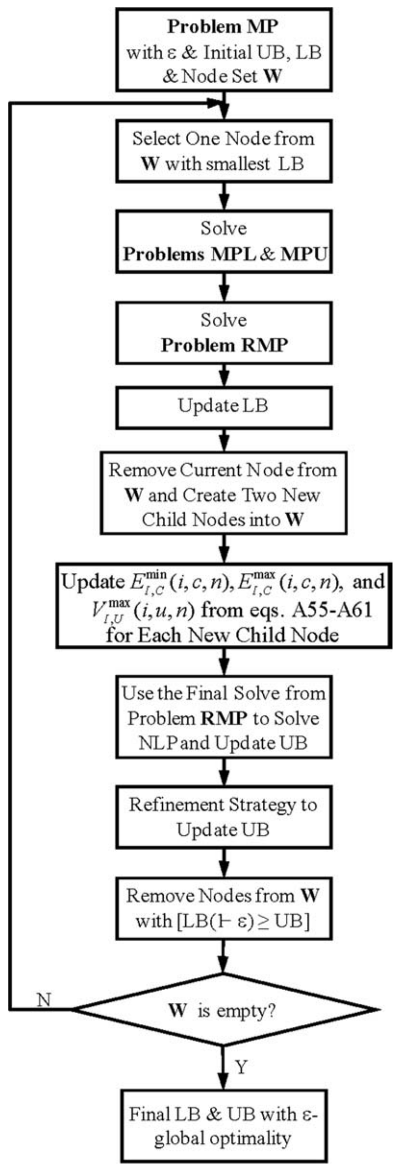

Figure 7. Flow chart of the extended branch and bound global optimization algorithm from Li et al.[3]

Table 3. Model and Solution Statistics for Example 2

	Nominal Solution	Robust Solution for Bounded Uncertainty	Robust Solution for Normal Distribution
Event points	3	3	3
GR	2	4	3
Binary variables	160	160	160
Continuous variables	1189	2865	2577
Constraints	5258	9076	8884
Bilinear terms	192	576	576
Obj (UB, K$)	−4795.037	−4780.727	−4789.280
LB (K$)	−4884.673	−4868.617	−4882.344
Gap (%)	1.84	1.81	1.92
CPU time (s)	4063	51600	5630

$Y = 0.50$, $\varepsilon = 0.02$.

(ROMU) **Max** z_obbt

s.t. Eqs. A1–A14, A17–A43 in **Appendix A**

Eqs. B3–B4 in **Appendix B**

Eqs. 25a, b, 26a, b, 27a, b, or 30a, b for corresponding distribution

Eqs. 31–33, 36–45, 46–49, and 50–51

Eqs. A45–A54 and Eqs. A55–A61 in **Appendix A**

$0 \le X(p,i,n), Y(i,u,n) \le 1$

where z_obbt becomes $V_I(i,n)$, $E_{I,C}(i,c,n)$, and $V_{I,U}(i,u,n)$ respectively to update $V_I^{\min}(i,n)$, $V_I^{\max}(i,n)$, $E_{I,C}^{\min}(i,c,n)$, $E_{I,C}^{\max}(i,c,n)$, and $V_{I,U}^{\max}(i,u,n)$ accordingly. The entire procedure of the branch and bound global optimization algorithm for ROM is illustrated in Figure 7.

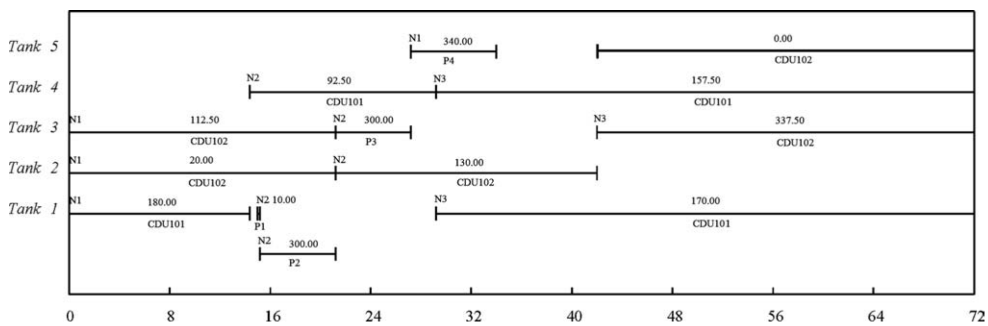

Figure 8. Robust schedule for Example 1 from model ROM with demand uncertainty following bounded uncertainty.

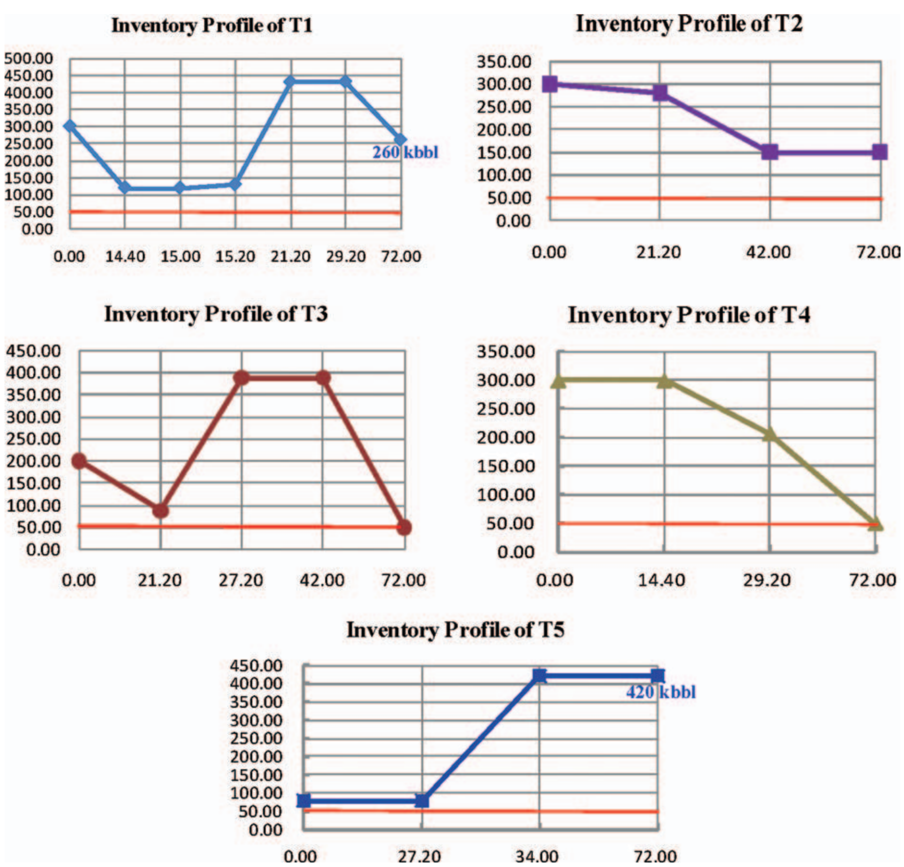

Figure 9. Inventory profiles of T1–T5 for Example 1 from the robust schedule in Figure 8.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

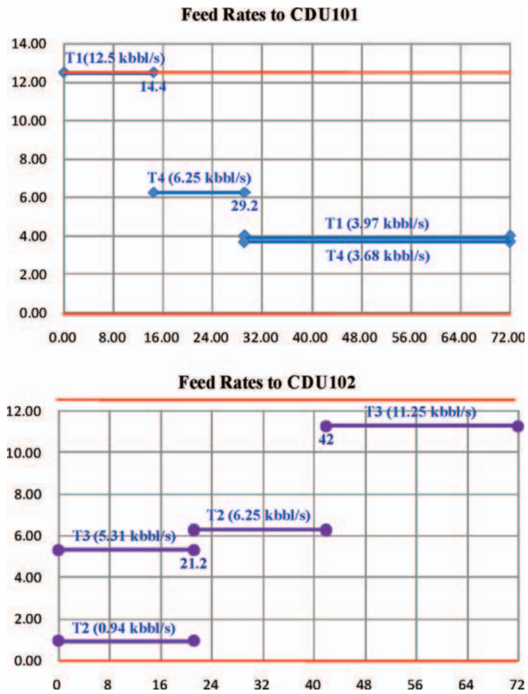

Figure 10. Feed rates to CDU101 and CDU102 for Example 1 from the robust schedule in Figure 8.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Computational Studies

The proposed deterministic robust counterpart optimization formulations are applied to two examples. The complete data for Example 2 are given in Supporting Information, Tables S1–S3. They are solved using GAMS 22.6/CPLEX 11.0.0 on Dell OPTIPLEX 960 of Intel ® Xeon^{TM} CPU 3.0 GHz with 2 GB RAM running Linux. The computational results are presented in Tables 2 and 3.

Example 1

Let us revisit Example 1 of Section "Motivating Example". The nominal schedule with nominal demands of 600 kbbl for both CDU101 and CDU102 is given in Figure 3. The objective is −$ 5631.707K.

Bounded Uncertainty. The demand parameters of both CDU101 and CDU102 are assumed to be uncertain and vary in a bounded interval [480,720] kbbl. When the demands of CDU101 and CDU102 increase to 705 kbbl, the nominal schedule is infeasible, as shown in Section "Motivating Example".

We use model ROM to generate the robust schedule (Figure 8) within 72.7 CPU seconds and with the objective of −$ 5614.974K, which deviates only by 0.298% from that of the nominal schedule. Note that the objective for the robust schedule is very close to that of nominal schedule. From the robust schedule (Figure 8), CDU101 processes 600 kbbl, and CDU102 processes 600 kbbl. Figures 9 and 10 illustrate the inventory profiles of T1–T5 and feed rates to CDU101 and CDU102, respectively. Note that T1 and T4 can feed CDU101, and T2, T3, and T5 can feed CDU102. From Figure 9, we observe that T1 has 210 kbbl additional crudes that can be charged to CDU101, although T4 reaches its minimum capacity at the end of scheduling horizon. Let us examine Figure 10, the feed rates of T1 and T4 to CDU101 are 3.97 kbbl/s and 3.68 kbbl/s, respectively, in [29.2, 72] h. As the maximum feed rate to CDU101 is 12.5 kbbl/s, the feed rate of T1 to CDU101 can increase to 8.82 kbbl/s during [29.2, 72] h. The maximum additional amount of crudes from T1 that can be charged to CDU101 is max[210kbbl, (8.82−3.97) × 42.8 kbbl] = 207.5 kbbl. Thus, any demand of CDU101 within [600, 720] kbbl can be satisfied by simply adjusting feed rate of T1 to CDU101 with the robust schedule in Figure 8. It is not difficult to conclude that any demand of CDU101 within [420, 600] kbbl can also be satisfied by simply adjusting feed rates of T1 and T4 to CDU101 with the robust schedule in Figure 8. A similar conclusion can be made for CDU102. In Supporting Information, Appendix S3, we show that the schedule in Figure 8 is feasible for any demand realization within the range [480, 720] kbbl by simply adjusting the feed rates from T1–T5 to CDU101 and CDU102.

Normal Distribution. We consider uncertainty with a normal distribution in the demands of both CDU101 and CDU102. The uncertainty level is $\varepsilon = 15\%$; the infeasibility tolerance is $\delta = 0\%$; and the reliability level $\kappa = 10\%$. We solve model ROM and obtain the robust schedule as shown in Figure 11 within 72.7 CPU seconds. The corresponding objective is −$ 5614.974K, which is only 0.298% different than the objective of the nominal schedule. From Figure 11, each CDU processes exact 600 kbbl of crudes. The inventory levels of T1–T5 and feed rates to CDUs are illustrated in Supporting Information, Figures S1 and S2. A similar analysis as bounded uncertainty can be done to conclude that the inventory levels in some tanks do not reach their maximum and minimum capacities and the feed rates from some tanks to CDUs do not reach maximum and minimum limits either. Therefore, demand uncertainty following normal distribution can be accommodated from the robust schedule (Figure 11) by simply adjusting feed rates

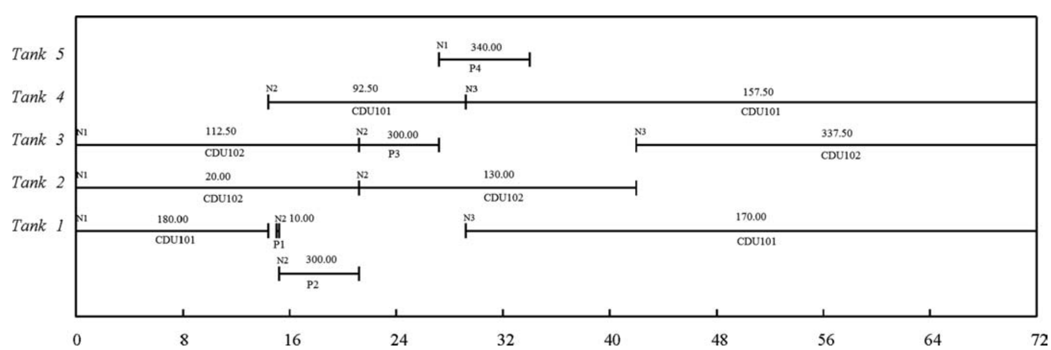

Figure 11. Robust schedule for Example 1 from model ROM with demand uncertainty following normal distribution.

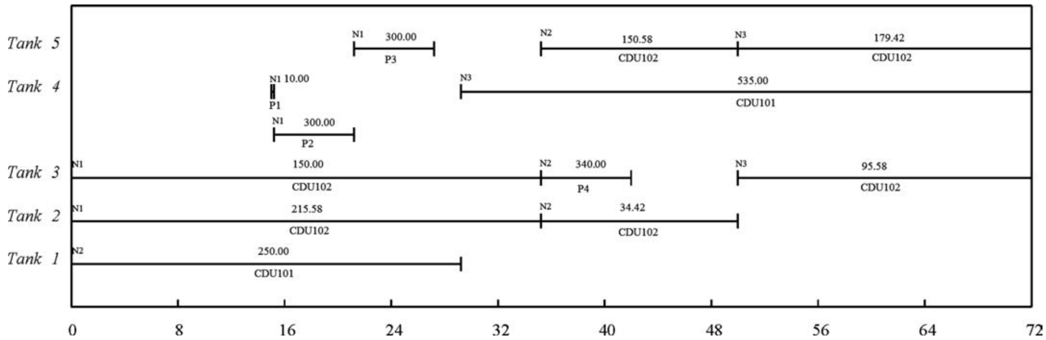

Figure 12. Robust schedule for Example 1 from ROM-CWR with demand uncertainty following bounded uncertainty.

to CDUs. By executing this robust schedule, we can ensure that the schedule is feasible by simply adjusting the feed rates from T1–T5 to CDU101 and CDU102 with a probability of 90% in the presence of the 15% uncertainty in the demands of both CDU101 and CDU102. It is interesting to note that the same schedule is obtained with bounded uncertainty.

The robust schedule for other known probability distributions such as uniform distribution and poisson distribution can also obtained from model ROM.

Remarks: We solve the deterministic robust counterpart model ROM-CWR (see Supporting Information, Appendix S2) and generate a schedule (Figure 12) with the objective of −$7248.618K. From Figure 12, it can be calculated that the total amount fed to CDU101 and CDU102 are 785.00 kbbl and 825.58 kbbl respectively, which are greater than the nominal demand (600 kbbl). It is not difficult to conclude that each CDU processes the maximum amount of crude mixture. As the objective of the model ROM-CWR is to maximize the total profit, the total amount fed to each CDU is maximized regardless of the specified demand for each CDU. Supporting Information, Figures S3 and S4 illustrate the inventory profiles of T1–T5 and feed rates to CDU101 and CDU102.

Example 2

This example is Example 21 from Li et al.[3,34] It is the largest industrial-scale problem involving one SBM pipeline, three jetties, eight storage tanks (T101–T108), and three CDUs (CDU101–CDU103). The scheduling horizon is about 336 hrs (i.e., 14 days). The nominal demands for CDU101, CDU102, and CDU103 are 1000 kbbl, 1000 kbbl, and 1000 kbbl, respectively. The nominal schedule is illustrated in Figure 13 with the objective of −$ 4795.037K. When the demands of CDU101, CDU102, and CDU103 increase to 1100 kbbl, the nominal schedule is infeasible.

Bounded Uncertainty. The demand parameters of CDU101, CDU102, and CDU103 are assumed to be uncertain and vary in a bounded interval [850, 1150] kbbl. The model ROM involves 160 binary variables, 2865 continuous variables, 9076 constraints, and 576 bilinear terms. The robust schedule (Supporting Information, Figure S5) from model ROM is obtained within 51,600 CPU seconds and the objective is −$ 4780.727K, which is guaranteed to be within 2% of global optimality. More importantly, the objective of −$ 4780.727K is only 0.257% different from that of nominal schedule. The inventory levels of T1–T8 and feed rates to CDUs are depicted in Supporting Information, Figures S6 and S7. Similar to Example 1, it can be concluded that the inventory levels in

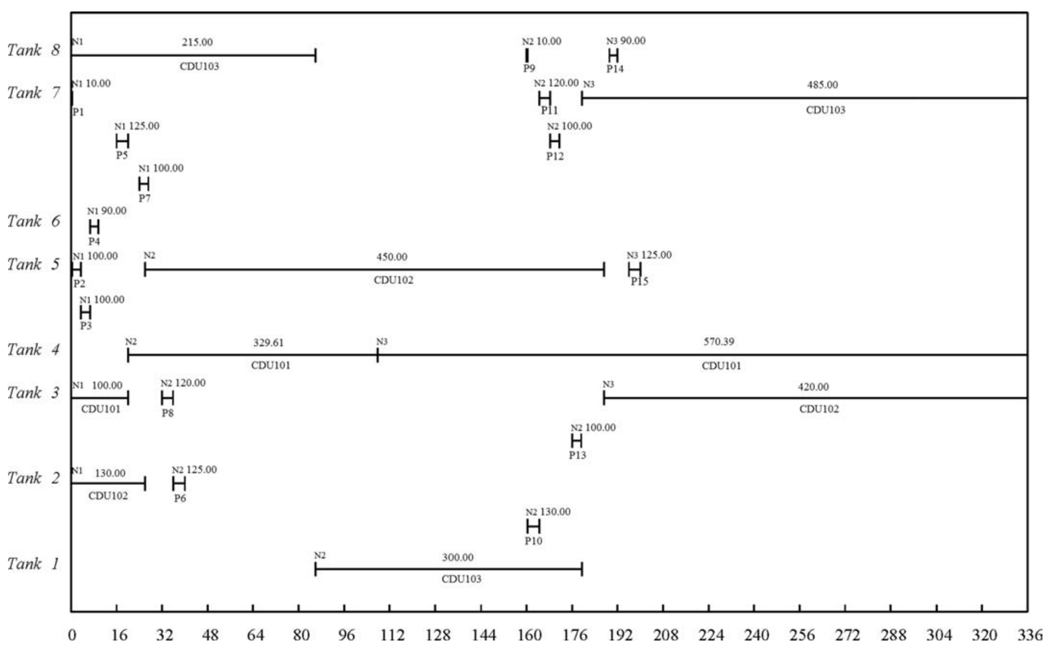

Figure 13. Nominal schedule for Example 2 from the proposed branch and bound global optimization algorithm of Li et al.[3]

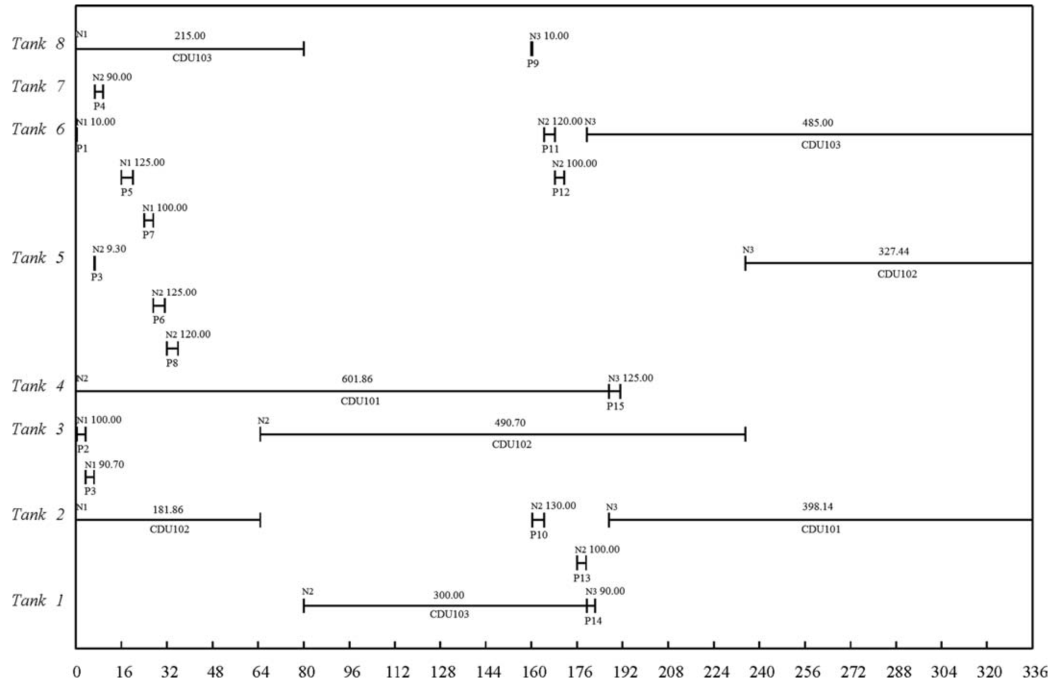

Figure 14. Robust schedule for Example 2 from model ROM with demand uncertainty following normal distribution.

some tanks do not reach their maximum and minimum capacities and the feed rates from some tanks to CDUs do not reach maximum and minimum limits either. Therefore, we can simply adjust feed rates to CDUs to accommodate demand uncertainty based on the robust schedule.

Normal Distribution. We also consider uncertainty with a normal distribution in the demands. The uncertainty level is $\varepsilon = 8\%$; the infeasibility tolerance is $\delta = 0\%$ and the reliability level $\kappa = 5\%$. In the model ROM, we have 160 binary variables, 2577 continuous variables, 8884 constraints, and 576 bilinear terms. The robust schedule (Figure 14) is obtained within 5630 CPU seconds and the objective is $-$\$ 4789.280K, which is guaranteed to be within 2% of global optimality. The objective function is also near to that of nominal schedule, only 0.120% different from that of nominal schedule. By executing this robust schedule (Figure 14), we can guarantee the schedule is feasible by simply adjusting the feed rates from T1–T8 to CDU101, CDU102, and CDU103 within with a probability of 95% in the presence of the 8% uncertainty in the demands of CDU101, CDU102, and CDU103. The inventory levels of T1–T8 and feed rates to CDUs are illustrated in Supporting Information, Figures S8 and S9.

Similar to Example 1, the schedules in Figure 14 and Supporting Information Figure S5 are more robust compared to the nominal schedule in Figure 13. The robust schedules for other known probability distributions such as uniform distribution and Poisson distribution can also be obtained from the model ROM.

Conclusions

In this article, we addressed the problem of scheduling of crude oil operations under demand uncertainty. The novel unit-specific event-based continuous-time MINLP formulation developed by Li et al.[3] and the robust optimization framework developed by Lin et al.[48] and Janak et al.[49] were successfully utilized and applied to develop robust optimization models, where a new approach was proposed to convert demand equality constraints to inequalities. The branch and bound global optimization algorithm proposed by Li et al.[3] was successfully extended to solve the deterministic robust counterpart optimization model. The computational results show that the generated schedule is more robust than the nominal schedule. In the future, we will extend the robust optimization framework to address other forms of uncertainty in ship arrival, quality specification and some economic coefficients.

Acknowledgments

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Appendix A: Deterministic Mathematical Formulation of Li et al.[3]

Notation

Sets

VP = VLCC parcels
JP = jetty parcels
$\mathbf{S}_{F,I}$ = set of pairs (tank i, event point n) that the concentration of tank i is the same as its initial composition during the possible event point n
$\mathbf{S}_{P,I}$ = set of pairs (parcel p, tank i) that tank i can receive parcel p
$\mathbf{S}_{I,C}$ = set of pairs (tank i, crude c) that tank i can hold crude c
$\mathbf{S}_{I,U}$ = set of pairs (tank i, CDU u) that tank i can feed CDU u
$\mathbf{S}_{U,C}$ = set of pairs (CDU u, crude c) that CDU u can process crude c

Parameters

Δn = 1 if a parcel is unloaded in multiple event points
C_{IVS} = inventory cost ($/unit/h)
C_{PEN} = safety stock penalty ($/unit/h)
$C_{\mathrm{PROF}}(c)$ = marginal profit ($/unit volume) from crude c
C_{SEA} = demurrage or sea-waiting cost
C_{SET} = cost (k$) per changeover
C_{ULD} = unloading cost ($/hr)
$D_U^{\min}(u)$ = minimum allowable crude processing rate of CDU u
$D_U^{\max}(u)$ = maximum allowable crude processing rate of CDU u
$D(u)$ = demand of each CDU u
$E_{I,C}^{\min}(i,c)$ = lower limit on the composition of crude c in tank i
$E_{I,C}^{\min}(i,c,n)$ = lower limit on the composition of crude c in tank i at event point n
$E_{I,C}^{\max}(i,c)$ = upper limit on the composition of crude c in tank i
$E_{I,C}^{\max}(i,c,n)$ = upper limit on the composition of crude c in tank i at event point n
$E_P(p,c)$ = fraction of crude c in parcel p
$E_U^{\min}(u,c)$ = minimum allowable composition of crude c in feed to CDU u
$E_U^{\max}(u,c)$ = maximum allowable composition of crude c in feed to CDU u
$e_C(c,k)$ = index of property k in crude c
$e_U^{\min}(u,k)$ = minimum allowable index of property k in CDU u
$e_U^{\max}(u,k)$ = maximum allowable index of property k in CDU u
$F_{I,U}^{\min}(i,u)$ = minimum feeding rate of crude from tank i to CDU u
$V_{I,U}^{\min}(i,u,n)$ = minimum feeding amount of crudes from tank i to CDU u at event point n
$F_{I,U}^{\max}(i,u)$ = maximum feeding rate of crude from tank i to CDU u
$V_{I,U}^{\max}(i,u,n)$ = maximum feeding amount of crudes from tank i to CDU u at event point n
$F_{P,I}^{\min}(p,i)$ = minimum unloading rate of crude from parcel p to tank i
$F_{P,I}^{\max}(p,i)$ = maximum unloading rate of crude from parcel p to tank i
$V_{P,I}^{\max}(p,i,n)$ = maximum unloading amount of crude from parcel p to tank i at event point n
H = scheduling horizon
SS = desired safety stock
ST = minimum time for crude settling and brine removal
$T_{\mathrm{ARR}}(p)$ = expected arrival time of parcel p
$V_I^{\mathrm{init}}(i)$ = initial crude volume in tank i
$V_{I,C}^{\mathrm{init}}(i,c)$ = initial amount of crude c in tank i
$V_P^{\mathrm{init}}(p)$ = initial crude volume of parcel p
$T_{\mathrm{ULD}}^{\min}(v)$ = stipulated departure time in the logistics contract for each vessel v
$V_I^{\min}(i)$ = minimum allowable crude inventory in tank i
$V_I^{\min}(i,n)$ = minimum allowable crude inventory in tank i at event point n
$V_I^{\max}(i)$ = maximum allowable crude inventory in tank i
$V_I^{\max}(i,n)$ = maximum allowable crude inventory in tank i at event point n

Binary variables

$X(p,i,n)$ = 1 if parcel p is unloaded to tank i during event point n
$Y(i,u,n)$ = 1 if tank i is charging CDU u during event point n

0–1 Continuous variables

$xe(p,n)$ = 1 if parcel p is completed at the end of event point n
$z(u,n)$ = 1 if a tank switch on CDU u takes place at the end of event n

Positive variables

$E_{I,C}(i,c,n)$ = composition of crude c in tank i at the end of event point n
SSP = average safety stock at the end of each event point
$T_B^{\mathrm{start}}(n)$ = start time of event point n on jetties
$T_B^{\mathrm{end}}(n)$ = end time of event point n on jetties
$T_{\mathrm{CW}}(v)$ = demurrage cost of vessel v
$T_{I,U}^{\mathrm{start}}(i,u,n)$ = start time that tank i feeds CDU u during event point n
$T_{I,U}^{\mathrm{end}}(i,u,n)$ = end time that tank i feeds CDU u during event point n
$T_P^{\mathrm{start}}(p)$ = start time for parcel p unloading
$T_P^{\mathrm{end}}(p)$ = end time for parcel p unloading
$T_{P,I}^{\mathrm{start}}(p,i,n)$ = start time that parcel p is unloaded to tank i during event point n
$T_{P,I}^{\mathrm{end}}(p,i,n)$ = end time that parcel p is unloaded to tank i during event point n
$T_U^{\mathrm{start}}(u,n)$ = start time of event point n on CDU u
$T_U^{\mathrm{end}}(u,n)$ = end time of event point n on CDU u
$V_{P,I}(p,i,n)$ = crude amount transferred from parcel p to tank i during event point n
$V_I(i,n)$ = crude volume in tank i at the end of event point n
$V_{I,C}(i,c,n)$ = volume of crude c in tank i at the end of event point n
$V_{I,U,C}(i,u,c,n)$ = amount of crude c fed from tank i to CDU u during event point n
$V_{I,U}(i,u,n)$ = amount of crude that tank i feeds to CDU u during event point n
$V_U(u,n)$ = total amount of crudes fed to CDU u during event point n

Constraints

The event point $(n+1)$ on unit m must start after the event point n on this unit m ends.

$$T_B^{\mathrm{start}}(n+1) \geq T_B^{\mathrm{end}}(n) \qquad \forall n \tag{A1a}$$

$$T_I^{\mathrm{start}}(i,n+1) \geq T_I^{\mathrm{end}}(i,n) \qquad \forall i,n \tag{A1b}$$

$$T_U^{\mathrm{start}}(u,n+1) \geq T_U^{\mathrm{end}}(u,n) \qquad \forall u,n \tag{A1c}$$

$$\sum_{i:(p,i)\in\mathbf{S}_{PI}} \sum_n X(p,i,n) = 1 \qquad \forall p, \Delta n = 0 \tag{A2a}$$

$$\sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n) \leq 1 \qquad \forall p,n,\Delta n = 1 \tag{A2b}$$

$$\sum_{\substack{p:(p,i)\in\mathbf{S}_{P,I} \\ p\in\mathbf{JP}}} X(p,i,n) \leq B \qquad \forall i,n,B>1 \tag{A3}$$

$$xe(p,n) \leq 2 - \sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n) - \sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n+1) \qquad \forall p, n<N, \Delta n = 1 \tag{A4a}$$

$$xe(p,n) \geq \sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n) - \sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n+1) \qquad \forall p, n<N, \Delta n = 1 \tag{A4b}$$

$$xe(p,n) \geq \sum_{i:(p,i)\in\mathbf{S}_{P,I}} X(p,i,n) \qquad \forall p, n<N, \Delta n = 1 \tag{A4c}$$

$$\sum_n xe(p,n) = 1 \qquad \forall p, \Delta n = 1 \tag{A5}$$

$$V_{P,I}(p,i,n) \geq F_{P,I}^{\min}(p,i) \cdot \left[T_{P,I}^{\mathrm{end}}(p,i,n) - T_{P,I}^{\mathrm{start}}(p,i,n)\right] \qquad \forall (p,i)\in\mathbf{S}_{P,I}, n \tag{A6a}$$

$$V_{P,I}(p,i,n) \leq F_{P,I}^{\max}(p,i) \cdot \left[T_{P,I}^{\mathrm{end}}(p,i,n) - T_{P,I}^{\mathrm{start}}(p,i,n)\right] \qquad \forall (p,i)\in\mathbf{S}_{P,I}, n \tag{A6b}$$

$$V_{P,I}(p,i,n) = V_P^{\text{init}}(p) \cdot X(p,i,n) \quad \forall (p,i) \in \mathbf{S}_{P,I}, \Delta n = 0 \tag{A7a}$$

$$V_{P,I}(p,i,n) \le V_{P,I}^{\max}(p,i,n) \cdot X(p,i,n) \quad \forall (p,i) \in \mathbf{S}_{P,I}, \Delta n = 1 \tag{A7b}$$

$$\sum_{i:(p,i)\in\mathbf{S}_{P,I}} \sum_n V_{P,I}(p,i,n) = V_P^{\text{init}}(p) \quad \forall p, \Delta n = 1 \tag{A8}$$

$$T_P^{\text{start}}(p) \le T_{P,I}^{\text{start}}(p,i,n) + H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, n \tag{A9a}$$

$$T_P^{\text{end}}(p) \ge T_{P,I}^{\text{end}}(p,i,n) - H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, n \tag{A9b}$$

$$T_P^{\text{start}}(p+1) \ge T_P^{\text{end}}(p) \quad \forall p \in \mathbf{VP} \tag{A10}$$

$$T_P^{\text{start}}(p') \ge T_P^{\text{end}}(p) \quad \forall p, p' \in \mathbf{JP}, T_{ARR}(p) < T_{ARR}(p'), B = 1 \tag{A11}$$

$$\sum_{c:(i,c)\in\mathbf{S}_{I,C}} V_{I,C}(i,c,n) = V_I(i,n) \quad \forall i, n \tag{A12}$$

$$V_I(i,n) \cdot E_I^{\min}(i,c) \le V_{I,C}(i,c,n) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n \tag{A13a}$$

$$V_{I,C}(i,c,n) \le V_I(i,n) \cdot E_I^{\max}(i,c) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n \tag{A13b}$$

$$V_{I,U,C}(i,u,c,n) = E_I^{\text{init}}(i,c) \cdot V_{I,U}(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U}, (i,c) \in \mathbf{S}_{I,C}, (i,n) \in \mathbf{S}_{F,I}, n \tag{A14}$$

$$V_{I,U,C}(i,u,c,n) = E_{I,C}(i,c,n-1) \cdot V_{I,U}(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U}, (i,c) \in \mathbf{S}_{I,C}, (i,n) \notin \mathbf{S}_{F,I}, n \tag{A15}$$

$$V_{I,C}(i,c,n) = E_{I,C}(i,c,n) \cdot V_I(i,n) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n \tag{A16}$$

$$X(p,i,n) + Y(i,u,n) \le 1 \quad \forall (p,i) \in \mathbf{S}_{P,I}, (i,u) \in \mathbf{S}_{I,U} \tag{A17}$$

$$\sum_{u:(i,u)\in\mathbf{S}_{I,U}} Y(i,u,n) \le 2 \quad \forall i, n \tag{A18a}$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}} Y(i,u,n) \le 2 \quad \forall u, n \tag{A18b}$$

$$V_{I,U}(i,u,n) \le F_{I,U}^{\max}(i,u) \cdot \left[T_{I,U}^{\text{end}}(i,u,n) - T_{I,U}^{\text{start}}(i,u,n)\right] \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A19a}$$

$$F_{I,U}^{\min}(i,u) \cdot \left[T_{I,U}^{\text{end}}(i,u,n) - T_{I,U}^{\text{start}}(i,u,n)\right] \le V_{I,U}(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A19b}$$

$$V_{I,U}(i,u,n) \le V_{I,U}^{\max}(i,u,n) \cdot Y(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A20}$$

$$V_{I,U}(i,u,n) = \sum_{c:(i,c)\in\mathbf{S}_{I,C}} V_{I,U,C}(i,u,c,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A21}$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}} V_{I,U}(i,u,n) = V_U(u,n) \quad \forall u, n \tag{A22}$$

$$D_U^{\min}(u)\left[T_U^{\text{end}}(u,n) - T_U^{\text{start}}(u,n)\right] \le V_U(u,n) \quad \forall u, n \tag{A23a}$$

$$V_U(u,n) \le D_U^{\max}(u)\left[T_U^{\text{end}}(u,n) - T_U^{\text{start}}(u,n)\right] \quad \forall u, n \tag{A23b}$$

$$V_U(u,n) \cdot E_U^{\min}(u,c) \le \sum_{i:(i,u)\in\mathbf{S}_{I,U}} V_{I,U,C}(i,u,c,n) \quad \forall (u,c) \in \mathbf{S}_{U,C} \tag{A24a}$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}} V_{I,U,C}(i,u,c,n) \le V_U(u,n) \cdot E_U^{\max}(u,c) \quad \forall (u,c) \in \mathbf{S}_{U,C} \tag{A24b}$$

$$e_U^{\min}(u,k) \cdot \sum_{i:(i,u)\in\mathbf{S}_{I,U}} V_{I,U}(i,u,n) \le \sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} e_C(c,k) \cdot V_{I,U,C}(i,u,c,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A25a}$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} e_C(c,k) \cdot V_{I,U,C}(i,u,c,n) \le e_U^{\max}(u,k) \cdot \sum_{i:(i,u)\in\mathbf{S}_{I,U}} V_{I,U}(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A25b}$$

$$e_U^{\min}(u,k) \cdot \left(\sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} \rho_c \cdot V_{I,U,C}(i,u,c,n)\right) \le \sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} e_C(c,k) \cdot \rho_c \cdot V_{I,U,C}(i,u,c,n) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A26a}$$

$$\sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} e_C(c,k) \cdot \rho_c \cdot V_{I,U,C}(i,u,c,n) \le e_U^{\max}(u,k) \cdot \left(\sum_{i:(i,u)\in\mathbf{S}_{I,U}} \sum_{c:(i,c)\in\mathbf{S}_{I,C}} \rho_c \cdot V_{I,U,C}(i,u,c,n)\right) \quad \forall (i,u) \in \mathbf{S}_{I,U} \tag{A26b}$$

$$\sum_n V_U(u,n) = D(u) \quad \forall u \tag{A27}$$

$$T_{P,I}^{\text{start}}(p,i,n+1) \ge T_{P,I}^{\text{end}}(p,i,n) \quad \forall (p,i) \in \mathbf{S}_{P,I}, n < N \tag{A28}$$

$$T_{P,I}^{\text{start}}(p,i',n+1) \ge T_{P,I}^{\text{end}}(p,i,n) - H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, (p,i') \in \mathbf{S}_{P,I}, n < N, \Delta n = 1 \tag{A29}$$

$$T_{P,I}^{\text{start}}(p',i,n+1) \ge T_{P,I}^{\text{end}}(p,i,n) - H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, (p',i) \in \mathbf{S}_{P,I}, p \ne p', n < N \tag{A30}$$

$$T_{P,I}^{\text{start}}(p,i',n+1) \le T_{P,I}^{\text{end}}(p,i,n) + H[2 - X(p,i,n) - X(p,i',n+1)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, (p,i') \in \mathbf{S}_{P,I}, n < N, \Delta n = 1 \tag{A31}$$

$$T_B^{\text{start}}(n) \le T_{P,I}^{\text{start}}(p,i,n) + H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{JP}, (p,i) \in \mathbf{S}_{P,I}, n < N, B > 1 \tag{A32a}$$

$$T_B^{\text{end}}(n) \geq T_{P,I}^{\text{start}}(p,i,n) - H[1 - X(p,i,n)] \quad \forall (p,i) \in \mathbf{JP}, (p,i) \in \mathbf{S}_{P,I}, n < N, B > 1 \tag{A32b}$$

$$T_{I,U}^{\text{start}}(i,u,n+1) \geq T_{I,U}^{\text{end}}(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U}, n < N \tag{A33}$$

$$T_{P,I}^{\text{start}}(p,i,n+1) \geq T_{I,U}^{\text{end}}(i,u,n) - H[1 - Y(i,u,n)] \quad \forall (p,i) \in \mathbf{S}_{P,I}, (i,u) \in \mathbf{S}_{I,U}, n < N \tag{A34}$$

$$T_{I,U}^{\text{start}}(i,u',n+1) \geq T_{I,U}^{\text{end}}(i,u,n) - H[1 - Y(i,u,n)] \quad \forall (i,u) \in \mathbf{S}_{I,U}, (i,u') \in \mathbf{S}_{I,U}, u \neq u', n < N \tag{A35}$$

$$T_U^{\text{start}}(u,n) \geq T_{I,U}^{\text{start}}(i,u,n) - H[1 - Y(i,u,n)] \quad \forall (i,u) \in \mathbf{S}_{I,U}, n \tag{A36a}$$

$$T_U^{\text{start}}(u,n) \leq T_{I,U}^{\text{start}}(i,u,n) + H[1 - Y(i,u,n)] \quad \forall (i,u) \in \mathbf{S}_{I,U}, n \tag{A36b}$$

$$T_U^{\text{end}}(u,n) \geq T_{I,U}^{\text{end}}(i,u,n) - H[1 - Y(i,u,n)] \quad \forall (i,u) \in \mathbf{S}_{I,U}, n \tag{A37a}$$

$$T_U^{\text{end}}(u,n) \leq T_{I,U}^{\text{end}}(i,u,n) + H[1 - Y(i,u,n)] \quad \forall (i,u) \in \mathbf{S}_{I,U}, n \tag{A37b}$$

$$T_{I,U}^{\text{start}}(i,u,n+1) \geq T_{P,I}^{\text{end}}(p,i,n) + ST \cdot X(p,i,n) - H\,[1 - X(p,i,n)] \ \forall (p,i) \in \mathbf{S}_{P,I}, (i,u) \in \mathbf{S}_{I,U}, n < N \tag{A38}$$

$$V_{I,C}(i,c,n) = V_{I,C}(i,c,n-1) + \sum_{p:(p,c)\in\mathbf{S}_{P,C}} V_{P,I}(p,i,n) \cdot E_P(p,c) - \sum_{u:(i,u)\in\mathbf{S}_{I,U}} V_{I,U,C}(i,u,c,n) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n > 1 \tag{A39a}$$

$$V_{I,C}(i,c,n) = V_{I,C}^{\text{init}}(i,c) + \sum_{p:(p,c)\in\mathbf{S}_{P,C}} V_{P,I}(p,i,n) \cdot E_P(p,c) - \sum_{u:(i,u)\in\mathbf{S}_{I,U}} V_{I,U,C}(i,u,c,n) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n = 1 \tag{A39b}$$

$$\sum_n \left[T_U^{\text{end}}(u,n) - T_U^{\text{start}}(u,n)\right] = H \quad \forall u \tag{A40}$$

$$z(u,n) \geq Y(i,u,n) - Y(i,u,n+1) \quad \forall (i,u) \in \mathbf{S}_{I,U}, n < N \tag{A41a}$$

$$z(u,n) \geq Y(i,u,n+1) - Y(i,u,n) \quad \forall (i,u) \in \mathbf{S}_{I,U}, n < N \tag{A41b}$$

$$T_{\text{CW}}(v) \geq C_{\text{SEA}} \cdot \left[T_P^{\text{end}}(p) - T_{\text{ARR}}(p) - T_{\text{ULD}}^{\min}(v)\right] \quad \forall (v,p) \in \mathbf{S}_{V,P}^L \tag{A42}$$

$$\text{SSP} \geq \text{SS} - \frac{\sum_i \left(\sum_n V_I(i,n) + V_I^{\text{init}}(i)\right)}{N+1} \tag{A43}$$

Objective

$$\text{PROFIT} = \sum_i \sum_u \sum_c \sum_n C_{\text{PROF}}(c) \cdot V_{I,U,C}(i,u,c,n) - \sum_v T_{\text{CW}}(v) - C_{\text{PEN}} \cdot \text{SSP} \cdot H - \sum_u \sum_n C_{\text{SET}} \cdot z(u,n) \tag{A44}$$

Hard Bounds

$$0 \leq xe(p,n) \leq 1 \quad \forall p,n \tag{A45}$$

$$0 \leq z(u,n) \leq 1 \quad \forall u,n \tag{A46}$$

$$SSP \leq SS \tag{A47}$$

$$E_{I,C}^{\min}(i,c,n) = 0, E_{I,C}^{\max}(i,c,n) = 1 \quad \forall (i,c) \in \mathbf{S}_{I,C}, n \tag{A48}$$

$$V_I^{\min}(i,n) = V_I^{\min}(i), V_I^{\max}(i,n) = V_I^{\max}(i) \quad \forall i,n \tag{A49}$$

$$T_{\text{ARR}}(p) \leq T_P^{\text{start}}(p) \leq H \quad \forall p \tag{A50a}$$

$$T_{\text{ARR}}(p) \leq T_P^{\text{end}}(p) \leq H \quad \forall p \tag{A50b}$$

$$T_{P,I}^{\text{start}}(p,i,n) \leq H, T_{P,I}^{\text{end}}(p,i,n) \leq H \quad \forall (p,i) \in \mathbf{S}_{P,I}, n \tag{A51a,b}$$

$$T_B^{\text{start}}(n) \leq H, T_B^{\text{end}}(n) \leq H \quad \forall n \tag{A52a,b}$$

$$T_{I,U}^{\text{start}}(i,u,n) \leq H, T_{I,U}^{\text{end}}(i,u,n) \leq H \quad \forall (i,u) \in \mathbf{S}_{I,U}, n \tag{A53a,b}$$

$$T_U^{\text{start}}(u,n) \leq H, T_U^{\text{end}}(u,n) \leq H \quad \forall u,n \tag{A54a,b}$$

Bound updates based on topology

First, the value of $V_I^{\max}(i,n)$ is given by

$$V_I^{\max}(i,n) = \begin{cases} V_I^{\text{init}}(i) + \sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I} \\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\leq V_I^{\max}(i)}} V_P^{\text{init}}(p) & \text{if } \Delta n = 0 \text{ and } n = 1 \\ \min\left\{V_I^{\max}(i), V_I^{\text{init}}(i) + \sum\limits_{p:(p,i)\in\mathbf{S}_{P,I}} V_P^{\text{init}}(p)\right\} & \text{if } \Delta n = 1 \text{ and } n = 1 \\ V_I^{\max}(i) & \text{if } n > 1 \end{cases} \quad \forall i \tag{A55}$$

Initialization: $E_{I,C}^{\min}(i,c,n) = 0$ and $E_{I,C}^{\max}(i,c,n) = 1$

$$E_{I,C}^{\min}(i,c,n)=\begin{cases}\max\left\{\dfrac{V_{I,C}^{\text{init}}(i,c)}{V_I^{\text{init}}(i)+\sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},\ p:(p,c)\notin\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i)}}V_P^{\text{init}}(p)},\dfrac{V_{I,C}^{\text{init}}(i,c)}{V_I^{\max}(i,n)}\right\} & \text{if } V_I^{init}(i)>0 \text{ and } \Delta n=0\\ \max\left\{\dfrac{V_{I,C}^{\text{init}}(i,c)}{V_I^{\text{init}}(i)+\sum\limits_{p:(p,i)\in\mathbf{S}_{P,I},\ p:(p,c)\notin\mathbf{S}_{P,C}}V_P^{\text{init}}(p)},\dfrac{V_{I,C}^{\text{init}}(i,c)}{V_I^{\max}(i,n)}\right\} & \text{if } V_I^{\text{init}}(i)>0 \text{ and } \Delta n=1\\ 0 & \text{if } V_I^{\text{init}}(i)=0\end{cases}\quad\forall(i,c)\in\mathbf{S}_{I,C},n=1 \tag{A56a}$$

$$E_{I,C}^{\min}(i,c,n)=\begin{cases}\dfrac{E_{I,C}^{\min}(i,c,n-1)\cdot V_I^{\min}(i,n-1)}{V_I^{\max}(i,n)} & \text{if } V_I^{\max}(i,n)>0\\ 0 & \text{if } V_I^{\max}(i,n)=0\end{cases}\quad\forall(i,c)\in\mathbf{S}_{I,C},n>1 \tag{A56b}$$

$$E_{I,C}^{\max}(i,c,n)=\begin{cases}\dfrac{\left[E_{I,C}^{\text{init}}(i,c)-1\right]\cdot\left[V_I^{\min}(i)\right]^{\sigma}}{\left[V_I^{\min}(i)+V_P^{\text{init}}(p)\right]^{\sigma}}+1 & \text{if } V_I^{\min}(i)>0\\ E_{I,C}^{\text{init}}(i,c) & \text{if } V_I^{\min}(i)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}=0\\ 1 & \text{if } V_I^{\min}(i)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}>0\end{cases}\quad\forall(i,c)\in\mathbf{S}_{I,C},n,\sigma=\sum_{\substack{p:(p,i)\in\mathbf{S}_{P,I}\\ p:(p,c)\in\mathbf{S}_{P,C}}}1 \tag{A57}$$

$$E_{I,C}^{\max}(i,c,n)=\begin{cases}\min\left\{\dfrac{V_{I,C}^{\text{init}}(i,c)+\sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}V_P^{\text{init}}(p)}{V_I^{\text{init}}(i)+\sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}V_P^{\text{init}}(p)},\dfrac{V_{I,C}^{\text{init}}(i,c)+V_I^{\max}(i,n)-V_I^{\text{init}}(i)}{V_I^{\max}(i,n)}\right\} & \text{if } V_I^{\text{init}}(i)>0\\ 1 & \text{if } V_I^{\text{init}}(i)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}>0\\ 0 & \text{if } V_I^{\text{init}}(i)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\text{init}}(i)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}=0\end{cases}$$
$$\forall(i,c)\in\mathbf{S}_{I,C},n=1,\Delta=0 \tag{A58a}$$

$$E_{I,C}^{\max}(i,c,n)=\begin{cases}\min\left\{\dfrac{V_{I,C}^{\text{init}}(i,c)+\sum\limits_{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}}V_P^{\text{init}}(p)}{V_I^{\text{init}}(i)+\sum\limits_{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}}V_P^{\text{init}}(p)},\dfrac{V_{I,C}^{\text{init}}(i,c)+V_I^{\max}(i,n)-V_I^{\text{init}}(i)}{V_I^{\max}(i,n)}\right\} & \text{if } V_I^{\text{init}}(i)>0\\ 1 & \text{if } V_I^{\text{init}}(i)=0 \text{ and } \max\limits_{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}}\left\{V_P^{\text{init}}(p)\right\}>0\\ 0 & \text{if } V_I^{\text{init}}(i)=0 \text{ and } \max\limits_{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}}\left\{V_P^{\text{init}}(p)\right\}=0\end{cases}$$
$$\forall(i,c)\in\mathbf{S}_{I,C},n=1,\Delta n=1 \tag{A58b}$$

$$E_{I,C}^{\max}(i,c,n)=$$
$$\begin{cases}\min\left\{\begin{array}{l}\dfrac{E_{I,C}^{\max}(i,c,n-1)\cdot V_I^{\min}(i,n-1)+\sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\min}(i,n-1)\le V_I^{\max}(i,n)}}V_P^{\text{init}}(p)}{V_I^{\min}(i,n-1)+\sum\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\min}(i,n-1)\le V_I^{\max}(i,n)}}V_P^{\text{init}}(p)},\\ \dfrac{E_{I,C}^{\max}(i,c,n-1)\cdot V_I^{\min}(i,n-1)+V_I^{\max}(i,n)-V_I^{\min}(i,n-1)}{V_I^{\max}(i,n)}\end{array}\right\} & \text{if } V_I^{\min}(i,n-1)>0\\ 1 & \text{if } V_I^{\min}(i,n-1)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\min}(i,n-1)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}>0\\ E_{I,C}^{\max}(i,c,n-1) & \text{if } V_I^{\min}(i,n-1)=0 \text{ and } \max\limits_{\substack{p:(p,i)\in\mathbf{S}_{P,I},p:(p,c)\in\mathbf{S}_{P,C}\\ V_P^{\text{init}}(p)+V_I^{\min}(i,n-1)\le V_I^{\max}(i,n)}}\left\{V_P^{\text{init}}(p)\right\}=0\end{cases}$$
$$\forall(i,c)\in\mathbf{S}_{I,C},n>1,\Delta n=0 \tag{A59a}$$

$$E_{I,C}^{\max}(i,c,n) = \begin{cases} \min\left\{ \dfrac{E_{I,C}^{\max}(i,c,n-1)\cdot V_I^{\min}(i,n-1) + \sum\limits_{p:(p,i)\in \mathbf{S}_{P,I}, p:(p,c)\in \mathbf{S}_{P,C}} V_P^{\text{init}}(p)}{V_I^{\min}(i,n-1) + \sum\limits_{p:(p,i)\in \mathbf{S}_{P,I}, p:(p,c)\in \mathbf{S}_{P,C}} V_P^{\text{init}}(p)}, \dfrac{E_{I,C}^{\max}(i,c,n-1)\cdot V_I^{\min}(i,n-1) + V_I^{\max}(i,n) - V_I^{\min}(i,n-1)}{V_I^{\max}(i,n)} \right\} & \text{if } V_I^{\min}(i,n-1) > 0 \\ 1 & \text{if } V_I^{\min}(i,n-1) = 0 \text{ and } \max\limits_{p:(p,i)\in \mathbf{S}_{P,I}, p:(p,c)\in \mathbf{S}_{P,C}} \left\{V_P^{\text{init}}(p)\right\} > 0 \\ E_{I,C}^{\max}(i,c,n-1) & \text{if } V_I^{\min}(i,n-1) = 0 \text{ and } \max\limits_{p:(p,i)\in \mathbf{S}_{P,I}, p:(p,c)\in \mathbf{S}_{P,C}} \left\{V_P^{\text{init}}(p)\right\} = 0 \end{cases}$$

$$\forall (i,c) \in \mathbf{S}_{I,C}, n > 1, \Delta n = 1 \quad \text{(A59b)}$$

We take the maximum value among Eqs. A57–A59 as final $E_{I,C}^{\max}(i,c,n)$.

The value of $V_{P,I}^{\max}(p,i,n)$ is computed by

$$V_{P,I}^{\max}(p,i,n) = \min\left\{V_{P,I}^{\max}(p,i)\cdot H, V_p^{\text{init}}\right\} \qquad \forall (p,i) \in \mathbf{S}_{P,I}, n \quad \text{(A60)}$$

The values of $V_{I,U}^{\min}(i,u,n)$ and $V_{I,U}^{\max}(i,u,n)$ are given by:

$$\begin{cases} V_{I,U}^{\min}(i,u,n) = 0 & \forall n \\ V_{I,U}^{\max}(i,u,n) = \min\left\{F_{I,U}^{\max}(i,u)\cdot H, V_I^{\text{init}} - V_I^{\min}(i,n), V_I^{\max}(i) - V_I^{\min}(i)\right\} & \text{if } n = 1 \\ V_{I,U}^{\max}(i,u,n) = \min\left\{\begin{matrix} F_{I,U}^{\max}(i,u)\cdot H, V_I^{\max}(i,n-1) - V_I^{\min}(i,n) \\ V_I^{\max}(i) - V_I^{\min}(i) \end{matrix}\right\} & \text{if } n > 1 \end{cases} \quad \forall (i,u) \in \mathbf{S}_{I,U} \quad \text{(A61)}$$

The entire nonconvex MINLP problem denoted as **MP** is presented as follows:

$$\begin{aligned} (\mathbf{MP}) \quad & \mathbf{Min} \quad -\text{PROFIT} \\ & s.t. \quad \text{Eqs. A1}-\text{A43} \\ & \qquad\ \ \text{Eqs. A45}-\text{A54 and Eqs. A55}-\text{A61} \end{aligned}$$

The resulting mathematical model **MP** is a nonconvex MINLP and the sources of nonconvexities are the distinct bilinear terms (i.e., Eqs. A15 and A16) presented as follows:

$$V_{I,U,C}(i,u,c,n) = E_{I,C}(i,c,n-1)\cdot V_{I,U}(i,u,n)$$
$$\forall (i,u) \in \mathbf{S}_{I,U}, (i,c) \in \mathbf{S}_{I,C}, (i,n) \notin \mathbf{S}_{F,I}, n \quad \text{(A15')}$$

$$V_{I,C}(i,c,n) = E_{I,C}(i,c,n)\cdot V_I(i,n) \quad \forall (i,c) \in \mathbf{S}_{I,C}, n \quad \text{(A16')}$$

Appendix B: Branch and Bound Global Optimization Algorithm

Piecewise-Linear Underestimators

In the branch and bound global optimization algorithm, variables $E_{I,C}(i,c,n)$ are uniformly partitioned and the **nf4r** formulation[30] is used to piecewise-underestimate each of the bilinear terms in the model. The details about **nf4r** formulation can be found in the paper of Gounaris et al.[30] and Misener and Floudas.[25]

The relaxation of the bilinear terms (Eqs. A15,A16) using the **nf4r** formulation is given by,

$$\begin{cases} V_{I,U,C}(i,u,c,n) \ge E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) + \sum\limits_{\text{gr}=1}^{\text{GR}} E_{I,C}(i,c,n-1,\text{gr}-1)\cdot \Delta V_{I,U}(i,u,c,n,\text{gr}) \\ V_{I,U,C}(i,u,c,n) \ge E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) + \\ \sum\limits_{\text{gr}=1}^{\text{GR}} \left\{E_{I,C}(i,c,n-1,\text{gr})\cdot \left[\Delta V_{I,U}(i,u,c,n,\text{gr}) - (V_{I,U}^{\max}(i,u,n) - V_{I,U}^{\min}(i,u,n))\cdot \lambda_{I,C}(i,c,n-1,\text{gr})\right]\right\} \\ V_{I,U,C}(i,u,c,n) \le E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) + \sum\limits_{\text{gr}=1}^{\text{GR}} E_{I,C}(i,c,n-1,\text{gr})\cdot \Delta V_{I,U}(i,u,c,n,\text{gr}) \\ V_{I,U,C}(i,u,c,n) \le E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) + \\ \sum\limits_{\text{gr}=1}^{\text{GR}} \left\{E_{I,C}(i,c,n-1,\text{gr}-1)\cdot \left[\Delta V_{I,U}(i,u,c,n,\text{gr}) - (V_{I,U}^{\max}(i,u,n) - V_{I,U}^{\min}(i,u,n))\cdot \lambda_{I,C}(i,c,n-1,\text{gr})\right]\right\} \end{cases}$$

$$\forall (i,u) \in \mathbf{S}_{I,U}, (i,c) \in \mathbf{S}_{I,C}, n \quad \text{(B1)}$$

$$\left\{\begin{array}{l} V_{I,C}(i,c,n) \geq E_{I,C}(i,c,n)\cdot V_I^{\min}(i,n) + \sum_{\mathrm{gr}=1}^{\mathrm{GR}} E_{I,C}(i,c,n,\mathrm{gr}-1)\cdot \Delta V_I(i,c,n,\mathrm{gr}) \\ V_{I,C}(i,c,n) \geq E_{I,C}(i,c,n)\cdot V_I^{\max}(i,n) + \\ \sum_{\mathrm{gr}=1}^{\mathrm{GR}} \left\{E_{I,C}(i,c,n,\mathrm{gr})\cdot \left[\Delta V_I(i,c,n,\mathrm{gr}) - (V_I^{\max}(i,n) - V_I^{\min}(i,n))\cdot \lambda_{I,C}(i,c,n,\mathrm{gr})\right]\right\} \\ V_{I,C}(i,c,n) \leq E_{I,C}(i,c,n)\cdot V_I^{\min}(i,n) + \sum_{\mathrm{gr}=1}^{\mathrm{GR}} E_{I,C}(i,c,n,\mathrm{gr})\cdot \Delta V_I(i,c,n,\mathrm{gr}) \\ V_{I,C}(i,c,n) \leq E_{I,C}(i,c,n)\cdot V_I^{\max}(i,n) + \\ \sum_{\mathrm{gr}=1}^{\mathrm{GR}} \left\{E_{I,C}(i,c,n,\mathrm{gr}-1)\cdot \left[\Delta V_I(i,c,n,\mathrm{gr}) - (V_I^{\max}(i,n) - V_I^{\min}(i,n))\cdot \lambda_{I,C}(i,c,n,\mathrm{gr})\right]\right\} \end{array}\right. \quad \forall (i,c)\in \mathbf{S}_{I,C}, n \qquad \text{(B2)}$$

When the number of grid points is equal to 2, the **nf4r** formulation is reduced to McCormick convex and concave envelopes as follows,

$$V_{I,U,C}(i,u,c,n) \left\{\begin{array}{l} \geq E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) + E_{I,C}^{\min}(i,c,n-1)\cdot V_{I,U}(i,u,n) - E_{I,C}^{\min}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) \\ \leq E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) + E_{I,C}^{\max}(i,c,n-1)\cdot V_{I,U}(i,u,n) - E_{I,C}^{\max}(i,c,n-1)\cdot V_{I,U}^{\min}(i,u,n) \\ \leq E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) + E_{I,C}^{\min}(i,c,n-1)\cdot V_{I,U}(i,u,n) - E_{I,C}^{\min}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) \\ \geq E_{I,C}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) + E_{I,C}^{\max}(i,c,n-1)\cdot V_{I,U}(i,u,n) - E_{I,C}^{\max}(i,c,n-1)\cdot V_{I,U}^{\max}(i,u,n) \end{array}\right. \quad \forall (i,u)\in \mathbf{S}_{I,U}, (i,c)\in \mathbf{S}_{I,C}, n \qquad \text{(B3)}$$

$$V_{I,C}(i,c,n) \left\{\begin{array}{l} \geq E_{I,C}(i,c,n)\cdot V_I^{\min}(i,n) + E_{I,C}^{\min}(i,c,n)\cdot V_I(i,n) - E_{I,C}^{\min}(i,c,n)\cdot V_I^{\min}(i,n) \\ \leq E_{I,C}(i,c,n)\cdot V_I^{\min}(i,n) + E_{I,C}^{\max}(i,c,n)\cdot V_I(i,n) - E_{I,C}^{\max}(i,c,n)\cdot V_I^{\min}(i,n) \\ \leq E_{I,C}(i,c,n)\cdot V_I^{\max}(i,n) + E_{I,C}^{\min}(i,c,n)\cdot V_I(i,n) - E_{I,C}^{\min}(i,c,n)\cdot V_I^{\max}(i,n) \\ \geq E_{I,C}(i,c,n)\cdot V_I^{\max}(i,n) + E_{I,C}^{\max}(i,c,n)\cdot V_I(i,n) - E_{I,C}^{\max}(i,c,n)\cdot V_I^{\max}(i,n) \end{array}\right. \quad \forall (i,c)\in \mathbf{S}_{I,C}, n \qquad \text{(B4)}$$

The piecewise-linear relaxation of problem **MP** (see Appendix A) is denoted as problem (**RMP**) and defined as follows,

(**RMP**)	**Min**	−PROFIT
	s.t.	Eqs. A1−A14, A17−A43, and B1−B2 or B3−B4
		Eqs. A45−A54 and Eqs. A55−A61

Branching Strategy

After solving a relaxation of each node using the piecewise-linear underestimator, the variable $V_I(i,n)$ is selected for branching and branching on $V_I(i,n)$ where (i, n) contributes to the greatest discrepancy between the auxiliary and original problem variables:

$$\max_{i,n}\left\{\sum_{c:(i,c)\in \mathbf{S}_{I,C}} \left|\hat{V}_{I,C}(i,c,n) - \hat{E}_{I,C}(i,c,n)\cdot \hat{V}_I(i,n)\right|\right\} \quad \forall i,n \qquad \text{(B5)}$$

where, ($\hat{V}_I$, $\hat{V}_{I,C}$, and $\hat{E}_{I,C}$) are the optimal solutions generated from the piecewise-linear relaxation for a given node. The branching point on $V_I(i,n)$ is 10% away from its optimal solution:

$$V_I^{\substack{\text{branching}\\ \text{point}}}(i,n) = \left\{\begin{array}{ll} 1.1\cdot \hat{V}_I(i,n) & \text{if } 1.1\cdot \hat{V}_I(i,n) < \frac{1}{2}\left[V_I^{\min}(i,n) + V_I^{\max}(i,n)\right] \\ 0.9\cdot \hat{V}_I(i,n) & \text{if } 0.9\cdot \hat{V}_I(i,n) > \frac{1}{2}\left[V_I^{\min}(i,n) + V_I^{\max}(i,n)\right] \\ \frac{1}{2}\left[V_I^{\min}(i,n) + V_I^{\max}(i,n)\right] & \text{otherwise} \end{array}\right. \qquad \text{(B6)}$$

Solution Improvement Strategy

The refinement strategy from Li et al.[34] is used to improve the quality of each solution from Pool-2. Let **S** denote any solution from Pool-2. First, the compositions of tanks from **S** are extracted and fixed in the MINLP to get an MILP. A solution S1 to this MILP is a schedule with no composition discrepancy. Then, the binary variables from solution S1 is fixed to get a NLP. This alternating series of MILP and NLP continues, until the solutions of successive NLPs converge. All improved solutions for Pool-2 form another solution pool (denoted as Pool-3). The best solution in Pool-3 is selected as the final UB.

Optimality-Based Tightening Lower and Upper Bounds

The lower and UBs for variables $E_{I,C}(i,c,n)$, $V_I(i,n)$, and $V_{I,U}(i,u,n)$ can be further tighten by solving LP minimization and maximization problems with Eqs. A1–A44 and McCormick convex and concave envelopes (i.e., Eqs. B3 and B4) to relax the bilinear terms (i.e., Eqs. A15 and A16). The minimization and maximization problems can be stated as follows

$$
\begin{aligned}
(\textbf{MPL})\quad & \text{Min} \quad z\_obbt \\
& s.t. \quad \text{Eqs. A1}-\text{A14, and A17}-\text{A43,} \\
& \qquad \text{A45}-\text{A54, A55}-\text{A61, and B3}-\text{B4} \\
& \qquad 0 \le X(p,i,n), Y(i,u,n) \le 1
\end{aligned}
$$

$$
\begin{aligned}
(\textbf{MPU})\quad & \text{Max} \quad z\_obbt \\
& s.t. \quad \text{Eqs. A1}-\text{A14, and A17}-\text{A43, A45}-\text{A54,} \\
& \qquad \text{A55}-\text{A61, and B3}-\text{B4} \\
& \qquad 0 \le X(p,i,n), Y(i,u,n) \le 1
\end{aligned}
$$

where *z_obbt* becomes $V_I(i,n)$, $E_{I,C}(i,c,n)$, and $V_{I,U}(i,u,n)$, respectively, to update $V_I^{\min}(i,n)$, $E_{I,C}^{\min}(i,c,n)$ $V_I^{\max}(i,n)$, $E_{I,C}^{\max}(i,c,n)$, and $V_{I,U}^{\max}(i,u,n)$ accordingly.

Additional Strategy

When solving an MILP relaxation using the piecewise-linear underestimators at each node, the relative convergence of this MILP relaxation is adjusted based on the difference between the best lower and UBs in the tree. It is required to be less than some ratio denoted as γ (e.g., 0.05) of the current relative gap between the LB and UB bounds. In other words,

$$\text{Relative Gap} = \gamma \cdot \left| \frac{\text{UB} - \text{LB}}{\text{LB}} \right| \tag{B7}$$

Note that as the MILP relaxation at each node may not be solved to optimality, the "best estimate" is always considered as the results of each node in the branch and bound tree.

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